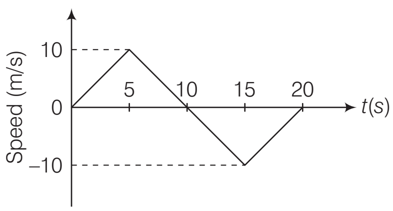
KERALA CEE

Engineering Entrance Exam

Solved Paper 2018

Physics

1. The one-dimensional motion of a point particle is shown in the figure. Select the correct statement.



(a) The total distance travelled by the particle is zero
(b) The total displacement of the particle is zero
(c) The maximum acceleration of the particle is $\frac{1}{2}\,\text{ms}^{-2}$
(d) The total distance travelled by the particle at the end of 10s is 100 m
(e) At the fifth second, the acceleration of the particle is $2\,\text{ms}^{-2}$

2. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum and g is the acceleration due to gravity. The length is measured using a meter scale which has 2000 divisions. If the measured value L is 50 cm, the accuracy in the determination of g is 1.1% and the time taken for 100 oscillations is 100 seconds, what should be the resolution of the clock (in milliseconds)?

(a) 1 (b) 2 (c) 5 (d) 0.25
(e) 0.1

3. From a circular card board of uniform thickness and mass M, a square disc of maximum possible area is cut. If the moment of inertia of the square with the axis of rotation at the centre and perpendicular to the plane of the disc is $\frac{Ma^2}{6}$, the radius of the circular card board is

(a) $\sqrt{2}a$ (b) $\frac{a}{\sqrt{2}}$ (c) $2a$ (d) $\frac{1}{2a}$
(e) $2\sqrt{2}a$

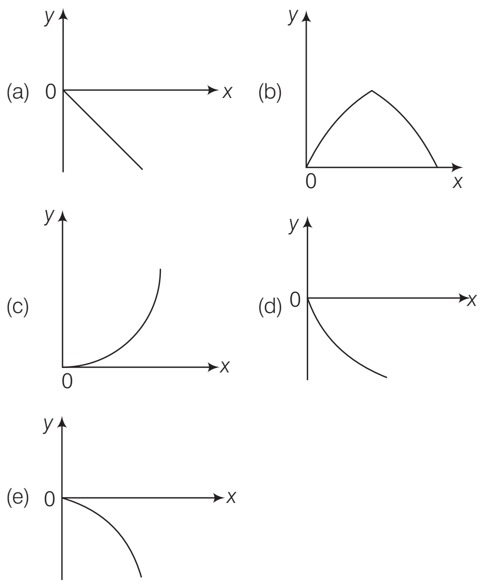
4. The length is measured using a vernier system whose main scale is 30 cm long with 600 divisions. If 19 divisions of the main scale coincide with 20 divisions of the vernier scale, then its least count is

(a) 0.25 cm (b) 0.025 cm
(c) 0.25 mm (d) 0.025 mm
(e) 0.0025 mm

5. A particle of mass m is moving along the X-axis under the potential $V(x) = \frac{kx^2}{2} + \frac{\lambda}{x}$, where k and x are positive constants of appropriate dimensions. The particle is slightly displaced from its equilibrium position. The particle oscillates with the angular frequency (ω) given by

(a) $3\frac{k}{m}$ (b) $3\frac{m}{k}$ (c) $\sqrt{\frac{k}{m}}$ (d) $\sqrt{3\frac{m}{k}}$
(e) $\sqrt{3\frac{k}{m}}$

6. Two particles of mass m and $2m$ have their position vectors as a function of time as $\mathbf{r}_1(t) = t\hat{\mathbf{i}} + t^3\hat{\mathbf{j}} + 2t^2\hat{\mathbf{k}}$ and $\mathbf{r}_1(t) = t\hat{\mathbf{i}} + t^3\hat{\mathbf{j}} + t^2\hat{\mathbf{k}}$ respectively (where t is the time). Which one of the following graphs represents the path of the centre of mass?



7. Two planets A and B have the same average density. Their radii R_A and R_B are such that $R_A : R_B = 3:1$. If g_A and g_B are the acceleration due to gravity at the surfaces of the planets, the $g_A : g_B$ equals

(a) 3 : 1 (b) 1 : 3 (c) 9 : 1 (d) 1 : 9
(e) $\sqrt{3} : 1$

8. The magnetic induction field has the dimensions of

(a) force
(b) force constant
(c) surface tension
(d) $\dfrac{\text{surface tension}}{\text{current}}$
(e) force constant λ current

9. Einstein was awarded the Nobel Prize for his work on

(a) photoelectric effect
(b) special theory of relativity
(c) brownian motion
(d) general theory of relativity
(e) quantum theory

10. A thin circular ring of mass m and radius R is rotating about its axis perpendicular to the plane of the ring with a constant angular velocity ω. Two point particles each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω/2. Then, the ratio m/M is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\sqrt{2}$
(e) $\frac{1}{\sqrt{2}}$

11. A body of mass $m = 1$ kg is moving in a medium and experiences a fractions force $F = -kv$, where v is the speed of the body. The initial speed is $v_0 = 10\ \text{ms}^{-1}$ and after 10s, its energy becomes half of initial energy. Then, the value of k is

(a) $10 \ln \sqrt{2}$ (b) $\ln \sqrt{2}$ (c) $\frac{\ln 2}{20}$ (d) $10 \ln 2$
(e) $\ln 2$

12. The position vector of the particle is $\mathbf{r}(t) = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}}$, where a and ω are real constants of suitable dimensions. The acceleration is

(a) perpendicular to the velocity
(b) parallel to the velocity
(c) directed away from the origin
(d) perpendicular to the position vector
(e) always along the direction of $\hat{\mathbf{i}}$

13. Some of the following equations are kinetic equations, where the symbols have their usual meaning. The work-energy theorem is represented by

(a) $v = u + at$ (b) $s = ut$
(c) $s = ut + \frac{1}{2}at^2$ (d) $v^2 = \frac{u^2}{2} + as$
(e) $v^2 = u^2 + 2as$

14. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following do not change with time?

(a) $\frac{aT}{v}$ (b) $aT + 2\pi v$
(c) $a^2T^2 + 4\pi^2v^2$ (d) aT
(e) vT

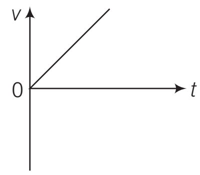
15. A rubber cord of density d, Young's modulus Y and length L is suspended vertically. If the cord extends by a length 0.5 L under its own weight, then L is

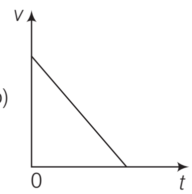
(a) $\frac{Y}{2dg}$ (b) $\frac{Y}{dg}$

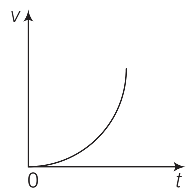
(c) $\frac{2Y}{dg}$ (d) $\frac{dg}{2Y}$

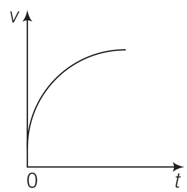
(e) $\frac{dg}{Y}$

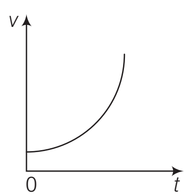
16. Which of the following graphs represents the speed v of a projectile as a function of time t?

(a)


(b)


(c)


(d)


(e)


17. A body P floats in water with half its volume immersed. Another body Q floats in a liquid of density 3/4th of the density of water with two-third of the volume immersed. The ratio of density of P to that of Q is

(a) 1 : 2 (b) 1 : 1

(c) 2 : 1 (d) 2 : 3

(e) 3 : 4

18. A pipe of 1 m length is closed at one end. Taking the speed of sound in air as 320 ms^{-1}, the air column in the pipe cannot resonate for the frequency (in Hz)

(a) 80 (b) 160 (c) 240 (d) 560

(e) 720

19. A wave pulse in a string is described by the equation $y_1 = \frac{5}{(3x-4t)^2+2}$ and another wave pulse in the same string is described by $y_2 = \frac{-5}{(3x+4t-6)^2+2}$. The values of y_1, y_2 and x are in metres and t in seconds.

Which of the following statement is correct?

(a) y_1 travels along $-x$-direction and y_2 along $+x$-direction

(b) Both y_1 and y_2 travel along $-x$-direction

(c) Both y_1 and y_2 travel along $+x$-direction

(d) At $x = 1$ m, y_1 and y_2 always cancel

(e) At time $t = 1$ s, y_1 and y_2 exactly cancel everywhere

20. The maximum transverse velocity and maximum transverse acceleration of a harmonic wave in a one-dimensional string are 1 ms^{-1} and 1 ms^{-2} respectively. The phase velocity if the wave is 1 ms^{-1}. The waveform is

(a) $\sin(x-t)$ (b) $\sin(2x-t)$

(c) $\sin(x-2t)$ (d) $\sin(x/2-t)$

(e) $\sin(x-t/2)$

21. Two particles A and B of same mass have their de-Broglie wavelengths in the ratio $X_A : X_B = K : 1$. Their potential energies $U_A : U_B = 1 : K^2$. The ratio of their total energies $E_A : E_B$ is

(a) $K^2 : 1$ (b) $1 : K^2$

(c) $K : 1$ (d) $1 : K$

(e) 1 : 1

22. A particle is moving along the X-axis such that its acceleration is proportional to the displacement from the equilibrium position and they are in the same direction. The displacement $x(t)$ is given by

(a) $\sin \omega t$, $\omega > 0$

(b) $\sin \omega t + \cos \omega t$, $\omega > 0$

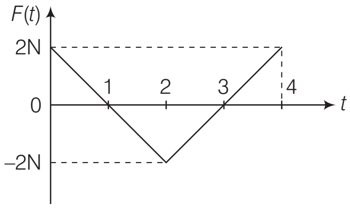
(c) $e^{\omega t}$, ω, >0

(d) $e^{\omega t} + \sin \omega t$, $\omega > 0$

(e) $e^{\omega t} + e^{\omega 2t}$, ω_1 and $\omega_2 > 0$

23. A block of mass 1 kg is free to move along the X-axis. It is at rest and from time $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$

varies with t as shown in figure. The kinetic energy of the block at $t = 4$ s is



(a) 1 J (b) 2 J (c) 3 J (d) 0 J
(e) 4 J

24. Consider a wire with density (d) and stress (σ). For the same density, if the stress increases 2 times, the speed of the transverse waves along the wire changes by

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\frac{1}{2}$
(e) 4

25. Two soap bubbles of radii 3 mm and 4 mm confined in vacuum coalesce isothermally to form a new bubble. The radius of the bubble formed (in mm) is

(a) 3 (b) 3.5 (c) 4 (d) 5
(e) 7

26. An oscillator circuit contains an inductor 0.05 H and a capacitor of capacity 80 μF. When the maximum voltage across the capacitor is 200 V, the maximum current (in amperes) in the circuit is

(a) 2 (b) 4 (c) 8 (d) 10
(e) 16

27. The displacement y of a particle, if given by $y = 4\cos^2(t/2)\sin(1000\,t)$. This expression may be considered to be a result of the superposition of how many simple harmonic motions?

(a) 4 (b) 3 (c) 2 (d) 5
(e) 6

28. A cylindrical tube, open at both the ends has fundamental frequency n. If one of the ends is closed, the fundamental frequency will become

(a) $\frac{n}{2}$ (b) $2n$ (c) n (d) $4n$
(e) $3n$

29. A uniform bar of mass m is supported by a pivot at its top about which the bar can swing like a pendulum. If a force F is applied perpendicular to the lower end of the bar as shown in figure, what is the value of F in order to hold the bar in equilibrium at an angle (θ) from the vertical

(a) $2mg\sin\theta$ (b) $mg\sin\theta$
(d) $mg\cos\theta$ (d) $\frac{mg}{2}\sin\theta$
(e) $\frac{mg}{2}\cos\theta$

30. A particle of rest mass m_0 is travelling, so that its total energy is twice its rest mass energy. It collides with another stationary particle of rest mass m_0 to form a new particle. What is the rest mass of the new particle?

(a) $\sqrt{6}m_0$ (b) $2m_0$
(c) $2\sqrt{3}m_0$ (d) $\sqrt{3}m_0$
(e) $3m_0$

31. The dimensions of ε_0 (permittivity in free space) is

(a) $ML^2T^4A^2$ (b) $ML^{-3}T^2A^2$
(c) $M^{-1}L^3T^4A^2$ (d) $ML^3T^2A^2$
(e) $M^{-1}L^{-3}T^4A^2$

32. The displacement of a wave is represented by $y = 0.6\times10^{-3}\sin(500t - 0.05x)$, where all the quantities are in their proper units. The maximum particle velocity (in ms^{-1}) of the medium is

(a) 0.5 (b) 0.03
(c) 0.150 (b) 0.75
(e) 0.3

33. The electric field of certain radiation is given by the equation
$E = 200\,\{\sin(4\pi\times10^{10})\,t + \sin(4\pi\times10^{15})t\}$ falls in a metal surface having work function 2.0 eV. The maximum kinetic energy (in eV) of the photoelectrons is [use Planck's constant $(h) = 6.63\times10^{-34}$ J-s and electron charge $(E) = 1.6\times10^{-19}$C]

(a) 3.3 (b) 4.3 (c) 5.3 (d) 6.3
(e) 7.3

34. The de-Broglie wavelength λ_n of the electron in the n^{th} orbit of hydrogen atom is

(a) inversely proportional to n
(b) proportional to n^2
(c) proportional to n
(d) inversely proportional to n^2
(e) inversely proportional to radius of the orbit in the n^{th} state

35. In a thermodynamic system, Q represents the energy transferred to or from a system by heat and W represents the energy transferred to or from a system by work

I. $Q > 0$ and $W = 0$ II. $Q < 0$ and $W = 0$
III. $W > 0$ and $Q = 0$ IV. $W < 0$ and $Q = 0$

Which of the above will lead to an increase in the internal energy of the system?

(a) I only (b) II only
(c) I and IV only (d) II and III only
(e) II and IV only

36. A cylinder closed at both ends is separated into two equal parts (45 cm each) by a piston impermeable to heat. Both the parts contain the same masses of gas at a temperature of 300 K and a pressure of 1 atm. How much the gas should be heated in one part of the cylinder to shift the piston by 5 cm and the pressure of the gas after shifting the piston?

(a) $T = 365$ K and $P = 1.125$ atm
(b) $T = 350$ K and $P = 1.125$ atm
(c) $T = 375$ K and $P = 2.125$ atm
(d) $T = 350$ K and $P = 2.125$ atm
(e) $T = 375$ K and $P = 1.125$ atm

37. Five moles of an ideal monoatomic gas with an initial temperature of 150°C expand and in the process absorb 1500 J of heat and does 2500 J of work. The final temperature of the gas in °C is (ideal gas constant, $R = 8.314\ JK^{-1}\ mol^{-1}$)

(a) 134 (b) 126 (c) 144 (d) 166
(e) 174

38. The temperature of an ideal gas is increased from 100 K to 400 K. If the rms speed of the gas molecule is v at 100 K, then at 400 K it becomes

(a) $2v$ (b) $4v$ (c) $0.5v$ (d) $0.25v$
(e) v

39. A uniform copper rod of 50 cm length is insulated on the sides and has its ends exposed to ice and steam respectively. If there is a layer of water 1 mm thick at each end, the temperature gradient (in °C m^{-1}) in the bar is (assume that the thermal conductivity of copper is 400 $Wm^{-1}K^{-1}$ and water is 0.4 $Wm^{-1}\ K^{-1}$)

(a) 60 (b) 40 (c) 50 (d) 55
(e) 65

40. A Carnot engine whose low temperature reservoir is at 350 K has an efficiency of 50%. It is desired to increase this to 60%. If the temperature of the low temperature reservoir remains constant, then the temperature of high temperature reservoir must be increased by how many degrees?

(a) 15 (b) 175 (c) 100 (d) 50
(e) 120

41. Two identical systems, with heat capacity at constant volume that varies as $C_v = bT^3$ (where b is a constant) are thermally isolated. Initially, one system is at a temperature 100 K and the other is at 200 K. The systems are then brought to thermal contact and the combined system is allowed to reach thermal equilibrium. The final temperature (in K) of the combined system will be

(a) 171 (b) 141 (c) 150 (d) 180
(e) 125

42. Water flows steadily through a horizontal pipe of a variable cross-section. If the pressure of the water is p at a point, where the speed of the flow is v, what is the pressure at another point, where the speed of the flow is $2v$? Let the density of water be

(a) $p + (3/2)\rho v^2$ (b) $p - 2\rho v^2$
(c) $p + 2\rho v^2$ (d) $p - 3\rho v^2$
(e) $p - (3/2)\rho v^2$

43. A soap bubble of radius r is blown upto form a bubble of radius $2r$ under isothermal conditions. If $\pi\sigma$ is the surface tension of soap solution, the energy spent in doing so is

(a) $6\pi\sigma r^2$ (b) $3\pi\sigma r^2$
(c) $24\pi\sigma r^2$ (d) $12\pi\sigma r^2$
(e) $9\pi\sigma r^2$

44. The mean momentum of a nucleon in a nucleus with mass number A varies as

(a) A^3 (b) A^2
(c) $A^{-2/3}$ (d) $A^{-1/3}$
(e) $A^{1/3}$

45. A decay chain of the nucleus $^{238}_{92}U$ involves eight α-decays and six β-decays. The final nucleus at the end of the process will be

(a) $Z = 76$; $A = 200$ (b) $Z = 84$; $A = 206$
(c) $Z = 84$; $A = 224$ (d) $Z = 82$; $A = 206$
(e) $Z = 82$; $A = 200$

46. A flat mirror revolves at a constant angular velocity making $n = 0.4$ revolutions per second. With what velocity (in ms^{-1}) will a light spot move along a spherical screen with a radius of 15 metres, if the mirror is at the centre of curvature of the screen?

(a) 37.7 (b) 60.3
(c) 68.7 (d) 75.4
(e) 90.4

47. A parallel beam of light of wavelength 4000 Å passes through a slit of width 5×10^{-3} m. The angular spread of the central maxima in the diffraction pattern is

(a) 1.6×10^{-3} rad (b) 1.6×10^{-4} rad
(c) 1.2×10^{-3} rad (d) 3.2×10^{-3} rad
(e) 3.2×10^{-4} rad

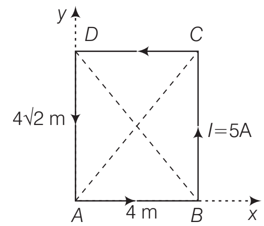
48. A wire made of aluminium having resistivity $\rho = 2.8 \times 10^{-8}$ m with a circular cross-section and has a radius of 2×10^{-3}m. A current of 5 A flows through the wire. If the voltage difference between the ends is 1 V, what is the length of the wire in metres?

(a) 50 (b) 60 (c) 90 (d) 120
(e) 110

49. When two capacitors are connected in parallel the resulting combination has capacitance 10 μF. The same capacitors when connected in series results in a capacitance 0.5 μF. The respective values of individual capacitors are

(a) 1.9 μF and 0.2 μF
(b) $(8 + 2\sqrt{5})$ μF and $(2 - 2\sqrt{5})$ μF
(c) $(5 + 2\sqrt{5})$ μF and $(5 - 2\sqrt{5})$ μF
(d) 12 μF and 1.7 μF
(e) 5 μF and 2 μF

50. A rectangular conducting loop of length $4\sqrt{2}$ m and breadth 4 m carrying a current of 5 A in the anti-clockwise direction is placed in the xy-plane. The magnitude of the magnetic induction field vector B at the intersection of the diagonal is (use $\mu_0 = 4 \times 10^{-7}$ NA^{-2})



(a) 1.2×10^{-6} T (b) 1.2×10^{-5} T
(c) 2.4×10^{-6} T (d) 2.4×10^{-5} T
(e) 1.2×10^{-7} T

51. Three point charges $4q$, Q and q are placed in a straight line of length L at points 0, $L/2$ and L respectively. The net force on charge q is zero. The value of Q is

(a) $4q$ (b) $-q$ (c) $-0.5q$ (d) $-2q$
(e) $3q$

52. A particle of charge Q moves with a velocity $\mathbf{v} = a\hat{\mathbf{i}}$ in a magnetic field $\mathbf{B} = b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ where a, b and c are constants. The magnitude of the force experienced by the particle is

(a) $Q(b + c)$ (b) zero
(c) $Q\sqrt{(ab)^2 + (ac)^2}$ (d) $Q\sqrt{(b^2 + c^2)}$
(e) $Qa(b - c)$

53. A point charge $+Q$ is held at rest at a point P. Another point charge $-q$, whose mass is m, moves at a constant velocity v in a circular orbit of radius R_1, around P. The work required to increase the radius of revolution of $-q$ from R_1, to another orbit R_2 is ($R_2 > R_1$)

(a) $\frac{Qq}{2}\left[\frac{1}{R_2} + \frac{1}{R_1}\right]$ (b) $\frac{Qq}{3}\left[\frac{1}{R_2}\right]$

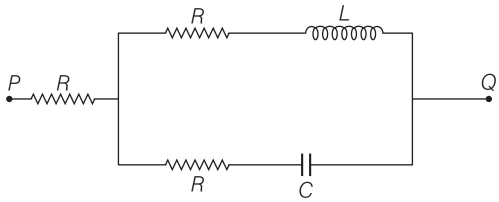
(c) $KQq\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$ (d) $-KQq\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$

(e) $2KQq\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

54. A voltage $V_{PQ} = V_0 \cos \omega t$ (where, V_0 is a real amplitude) is applied between the points P and Q in the network shown in the figure. The values of capacitance and inductance are

$$C = \frac{1}{\omega R\sqrt{3}} \quad \text{and} \quad L = \frac{R\sqrt{3}}{\omega}$$

Then, the total impedance between P and Q is



(a) 1.5 R (b) $2R$ (c) $3R$ (d) $4R$
(e) 2.5 R

55. Two particles A and B of same mass have their total energies E_A and E_B in the ratio $E_A : E_B = 1:2$. Their potential energies U_A and U_B are in the ratio $U_A : U_B = 1:2$. If λ_A and λ_B are their de-Broglie wavelengths, then $\lambda_A : \lambda_B$ is

(a) 1 : 2 (b) 2 : 1
(c) 1 : $\sqrt{2}$ (d) $\sqrt{2}$: 1
(e) 1 : 1

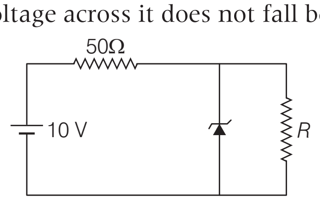
56. The electrical conductivity of a metal is

(a) directly proportional to the mean free path
(b) directly proportional to the mass of electron
(c) inversely proportional to the relaxation time
(d) inversely proportional to the mean free path
(e) directly proportional to the average speed of free electrons

57. A 2 MeV neutron is emitted in a fission reactor. If it looses half of its kinetic energy in each collision with a moderator atom, how many collisions must it undergo to achieve thermal energy of 0.039 eV?

(a) 20 (b) 26 (c) 30 (d) 42
(e) 48

58. The 6V Zener diode is shown in figure has negligible resistance and a knee current of 5 mA. The minimum value of R (in Ω) so that the voltage across it does not fall below 6 V is



(a) 40 (b) 60 (c) 72 (d) 80
(e) 120

59. An electron is moving with a velocity 2×10^6 m/s along positive x-direction in the uniform electric field of 8×10^7 V/m applied along positive y-direction. The magnitude and direction of a uniform magnetic field (in tesla) that will cause the electrons to move undeviated along its original path is

(a) 40 in – ve z-direction (b) 40 in + ve z-direction
(c) 4 in + ve z-direction (d) 4 in – ve z-direction
(e) 8 in + ve z-direction

60. What is the minimum thickness (in nm) of a soap film ($n = 1.3$) that results in constructive interference in reflected light if the film is illuminated with light whose wavelength in free space is 620 nm

(a) 100 (b) 120 (c) 160 (d) 240
(e) 180

61. Three variable Boolean expression $PQ + PQR + \overline{P}Q + P\overline{Q}R$ can be written as

(a) $\overline{Q} + \overline{P}R$ (b) $\overline{P} + \overline{Q}R$
(c) $Q + PR$ (d) $Q + \overline{P}R$
(e) $P + QR$

62. A prism is made up of material of refractive index $\sqrt{2}$. The angle of the prism is A. If the angle of minimum deviation is equal to the angle of the prism, the value of A is

(a) 30° (b) 45° (c) 60° (d) 75°
(e) 90°

63. Consider a cylindrical conductor of length L and area of cross-section A. The specific conductivity varies as $\sigma(x) = \sigma_0 \frac{L}{\sqrt{x}}$ where x is the distance along the axis of the cylinder from one of its ends. The resistance of the system along the cylindrical axis is

(a) $\frac{2\sqrt{L}}{3A\sigma_0}$ (b) $\frac{3\sqrt{L}}{2A\sigma_0}$ (c) $\frac{\sqrt{L}}{3A\sigma_0}$ (d) $\frac{2\sqrt{L}}{A\sigma_0}$
(e) $\frac{4\sqrt{L}}{3A\sigma_0}$

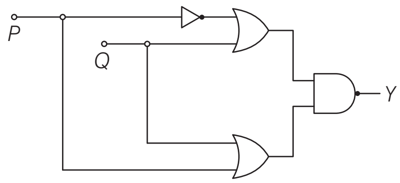
64. If the emission rate of blackbody at 0°C is R, then the rate of emission at 273°C is

(a) 2 R (b) 4 R (c) 8 R (d) 16 R
(e) 32 R

65. For any material, if R, T and A represent the reflection coefficient, transparent coefficient and absorption coefficient respectively, then for a blackbody which one of the following is true

(a) $R=1, T=0, A=0$ (b) $R=1, T=1, A=0$
(c) $R=0, T=1, A=1$ (d) $R=0, T=0, A=1$
(e) $R=0, T=1, A=0$

66. In the given circuit P and Q form the inputs. The output Y is



(a) $Y=\overline{P}$ (b) $Y=P\overline{Q}$
(c) $Y=P+Q$ (d) $Y=\overline{Q}$
(e) $Y=\overline{P}+Q$

67. A radio transmitter sends out 60 W of radiation. Assuming that the radiation is uniform on a sphere with the transmitter at its centre, the intensity (in W/m^2) of the wave at a distance 12 km is

(a) 5.33×10^{-8} (b) 3.33×10^{-6}
(c) 2.12×10^{-8} (d) 6.66×10^{-8}
(e) 3.33×10^{-8}

68. Consider a system of gas of a diatomic molecule in which the speed of sound at 0°C is 1260 ms^{-1}. Then, the molecular weight of the gas is (given, the gas constant R is 8.314 J/mol K)

(a) 2g (b) 2 mg
(c) 4 g (d) 10 g
(e) 20 g

69. A satellite is orbiting the Earth in a circular orbit of radius R. Which one of the following statements is true ?

(a) Angular momentum varies as $\frac{1}{\sqrt{R}}$
(b) Linear momentum varies as $\sqrt{R}$
(c) Frequency of revolution varies as $\frac{1}{R^2}$
(d) Kinetic energy varies as $\frac{1}{R}$
(e) Potential energy varies as R

70. The magnitude of a magnetic field at the centre of a circular coil of radius R, having N turns and carrying a current I can be doubled by changing

(a) I to $2I$ and N to $2N$ keeping R unchanged
(b) N to $N/2$ and keeping I and R unchanged
(c) N to $2N$ and R to $2R$ keeping I unchanged
(d) R to $2R$ and I to $2I$ keeping N unchanged
(e) I to $2I$ and keeping N and R unchanged

71. An alternating voltage $V=V_0\sin\omega t$ is applied across a circuit and as a result, a current $I=I_0\sin\left(\omega t+\frac{\pi}{2}\right)$ flows in it. The power consumed per cycle is

(a) I_0V_0 (b) $0.5I_0V_0$ (c) $0.7\,I_0V_0$ (d) $1.41I_0V_0$
(e) 0

72. An electromagnetic wave of intensity I is incident on a non-reflecting surface. If C is the speed of light in free space, then the ratio I/C is same as

(a) momentum (b) force
(c) pressure (d) pressure per unit area
(e) force x-area

Chemistry

73. Which element has the highest first ionisation potential ?

(a) N (b) Ne (c) He (d) H
(e) Li

74. Which statement(s) is (are) false for the periodic classification of elements?

(a) The properties of the elements are the periodic functions of their atomic numbers
(b) Non-metallic elements are lesser in number than the metallic elements
(c) The first ionisation energies of the elements along a period do not vary in a regular manner with increase in atomic number
(d) For transition elements, the d-electrons are filled monotonically with increase in atomic number
(e) Both (c) and (d)

75. The electronegativities of N, C, Si and P are in the order

(a) P < Si < C < N
(b) Si < P < N < C
(c) Si < P < C < N
(d) P < Si < N < C
(e) Difficult to predict

76. Gd(64) has __ unpaired electrons with sum of spin __.

(a) 7, 3.5
(b) 8, 3
(c) 6, 3
(d) 8, 4
(e) 9, 3.5

77. When SO_2 gas is passed into aqueous Na_2CO_3 the product(s) formed is (are)

(a) $NaHSO_4$
(b) Na_2SO_4
(c) $NaHSO_3$
(d) Na_2SO_3 and $NaHSO_3$
(e) $NaHSO_4$ and Na_2SO_4

78. Portland cement does not contain

(a) $CaSiO_4$
(b) $CaSiO_3$
(c) $Ca_3Al_2O_6$
(d) $Ca_3(PO_4)_2$
(e) Both (c) and (d)

79. $Al_2(SO_4)_3$ is used in the following but not

(a) as a coagulant in treating drinking water and sewage
(b) in plastics industry
(c) as a mordant in dyeing
(d) in paper industry
(e) Both (c) and (d)

80. Maximum number of covalent bonds formed by N and P are

(a) 3,5
(b) 3,6
(c) 3, 4, 5
(d) 3, 4, 6
(e) None of these

81. Consider the following statements concerning N_2H_4:

1. It is an exothermic compound.
2. It burns in air with the evolution of heat.
3. It has kinetic stability.
4. In reduces Fe^{3+} to Fe^{2+} in acidic medium.

Which of the following combination is correct?

(a) 2 and 3 are correct
(b) 1 and 2 are correct
(c) All are correct
(d) 3 and 4 are correct
(e) 2, 3 and 4 are correct

82. Consider the following species :

1. $[O_2]^{2-}$
2. $[CO]^+$
3. $[O_2]^+$

Among these sigma bond alone is present in

(a) 1 alone
(b) 2 alone
(c) 3 alone
(d) 1 and 2
(e) 1, 2 and 3

83. Select the correct option(s) for the following statements :

1. Cl_2O and ClO_2 are used as bleaching agents.
2. OCl^- salts are used as detergents.
3. OCl^{-1} disproportionates in alkaline medium.
4. BrO_3^- is oxidised in acidic medium.

(a) 1, 2, 3 correct
(b) 2, 3, 4 correct
(c) 1, 2, 4 correct
(d) 1, 3, 4 correct
(e) All are correct

84. When H_2O_2 is added to an acidified $K_2Cr_2O_7$ solution

(a) a green colour solution is obtained
(b) a yellow solution is obtained
(c) a blue-violet solution is obtained
(d) a green precipitate is formed
(e) a yellow precipitate is formed

85. Consider the following compounds :

1. $(NH_4)_2Cr_2O_7$
2. NH_4NO_2
3. NH_4VO_3
4. NH_4NO_3

Which compound(s) yield nitrogen gas upon heating?

(a) 1 and 2
(b) 2 and 3
(c) 3 and 4
(d) 1 and 4
(e) All

86. How many peroxy linkages are present in CrO_5?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

87. More than four bonds are made by how many elements in carbon family?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

88. The effective nuclear charge of an element with three valence electrons is 2.60. What is the atomic number of the element?

(a) 1 (b) 2
(c) 3 (d) 4
(e) 5

89. The elution sequence of a mixture of compounds containing chlorobenzene, anthracene and *p*-cresol developed of an alumina column using a solvent system of progressively increasing polarity is

(a) anthracene chlorobenzene *p*-cresol
(b) anthracene *p*-cresol chlorobenzene
(c) chlorobenzene *p*-cresol anthracene
(d) chlorobenzene anthracene *p*-cresol
(e) *p*-cresol anthracene chlorobenzene

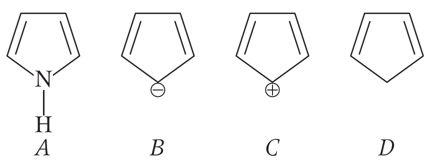
90. Number of constitutional isomers of alkane with formula C_6H_{14} is

(a) 3 (b) 2
(c) 5 (d) 10
(e) 8

91. Phenylacetylene on treatment with $HgSO_4/H_2SO_4$, H_2O produces

(a) acetophenone
(b) phenylacetaldehyde
(c) phenylacetic acid
(d) 1-phenylethanol
(e) 2- phenyletharol

92. Which of the following compounds are aromatic?



(a) *A, B*
(b) *A, B, C*
(c) *B, C*
(d) *B, C, D*
(e) *A, B, D*

93. Aromatic electrophilic substitution reaction that is reversible is

(a) nitration
(b) chlorination
(c) sulphonation
(d) alkylation
(e) acylation

94. Which one of the following statements is false?

(a) R and S configurations correspond to the enantiomers of an optically active compound
(b) The process of converting an optically active compound into a racemate is called racemisation
(c) A molecule containing a plane of symmetry can be optically active
(d) Optical isomers that are not enantiomers are called diastereoisomers
(e) All chiral objects are asymmetric

95. Neopentyl bromide, undergoes dehydrohalogenation to give alkenes even though it has no hydrogen. This is due to

(a) E2 mechanism
(b) E1 mechanism
(c) Rearrangement of carbocations by E1 mechanism
(d) E1cB mechanism
(e) E1 mechanism

96. The compound which does not lead to nitrile by substitution with NaCN/DMSO is

(a) benzyl chloride (b) ethyl chloride
(c) isopropyl chloride (d) chlorobenzene
(e) isobutyl chloride

97. Oxidation of 1 π alcohols to aldehydes is very successful for the alcohols like

(a) pent-2-yn-1-ol (b) 1-hexanol
(c) *n*-propyl alcohol (d) 1- pentanol
(e) 1-octanol

98. The compound that does not undergo halo form reaction is

(a) Acetaldehyde (b) Ethanol
(c) Acetone (d) Acetophenone
(e) Propiophenone

99. The halogen compound which will not react with phenol to give ethers is

(a) ethyl chloride (b) methyl chloride
(c) benzyl chloride (d) vinyl chloride
(e) allyl chloride

100. The weakest among the following acids is

(a) peroxyacetic acid
(b) acetic acid
(c) chloroacetic acid
(d) trichloroacetic acid
(e) propanoic acid

101. The nitrosation of N, N-dimethylaniline takes place through the attack of electrophile

(a) nitronium ion
(b) protonated nitrous acid
(c) nitrous acid
(d) nitrite ion
(e) nitrosonium ion

102. The nitrogeneous base present only in RNA is

(a) guanine (b) adenine
(c) cytosine (d) uracil
(e) thymine

103. Green fuel is the fuel obtained from

(a) bio-waste (b) metal waste
(c) plastic waste (d) chemical waste
(e) electronic waste

104. Barbiturates are potent

(a) hypnotics
(b) antimicrobials
(c) antacids
(d) antiseptics
(e) antiallergics

105. 1 mol of $FeSO_4$ (atomic weight of Fe is 55.84 g mol^{-1}) is oxidised to $Fe_2(SO_4)_3$. Calculate the equivalent weight of ferrous ion

(a) 55.84 (b) 27.92
(c) 18.61 (d) 111.68
(e) 83.76

106. Mass % of carbon in ethanol is

(a) 52 (b) 13
(c) 34 (d) 90
(e) 80

107. One mole of ethanol is produced reacting graphite, H_2 and O_2 together. The standard enthalpy of formation is −277.7 kJ mol^{-1}. Calculate the standard enthalpy of the reaction when 4 moles of graphite is involved

(a) −277.7 (b) −555.4
(c) −138.85 (d) −69.42
(e) −1110.8

108. Which of the following process best describes atomisation of $CH_4(g)$?

(a) Exothermic (b) Endothermic
(c) Non-spontaneous
(d) Spontaneous
(e) Both (b) and (c)

109. Consider the equilibrium $X_2 + Y_2 \rightleftharpoons P$. Find the stoichiometric coefficient of the P using the data given in the following table :

X_2/mol L^{-1}	Y_2/mol L^{-1}	P/mol L^{-1}
1.14×10^{-2}	0.12×10^{-2}	2.52×10^{-2}
0.92×10^{-2}	0.22×10^{-2}	3.08×10^{-2}

(a) 1 (b) 2
(c) 3 (d) 0.5
(e) 4

110. Which of the following can help predict the rate of a reaction if the standard Gibbs free energy of reaction (⇐Gπ) is known?

(a) Equilibrium constant
(b) ⇐H π
(c) ⇐U π
(d) Heat liberated during the course of reaction in calorimeter
(e) Both (a) and (b)

111. Calculate the molarity of a solution containing 5g of NaOH dissolved in the product of a H_2- O_2 fuel cell operated at 1 A curent for 595.1 hours.
(Assume 1F = 96500 C/mol of electrons and molecular weight of NaOH as 40 g mol^{-1})

(a) 0.05 M (b) 0.025 M
(c) 0.1 M (d) 0.075 M
(e) 1M

112. If 1 mol of NaCl solute is dissolved into the 1 kg of water, at what temperature will water boil at 1.013 bar?
(K_b of water is 0.52 K kg mot^{-1})

(a) 373.15 K (b) 373.67 K
(c) 374.19 K (d) 373.19 K
(e) 375 K

113. Consider the electrochemical reaction between Ag(s) and $Cl_2(g)$ electrodes in 1 L of 0.1 M KCl aqueous solution. Solubility product of AgCl is 1.8 λ10^{-10} and F = 96500 C/mol. At 1 ≥10^{-6} A current, calculate the time required to start observing the AgCl precipitation in the galvanic cell

(a) 173 s (b) 346 s
(c) 1.25 λ 10^6 s (d) 1.25 λ 10^5 s
(e) 101 s

114. The voltage of the cell consisting of Li (s) and $F_2(g)$ electrodes is 5.92 V at standard condition at 298 K. What is the voltage if the electrolyte consists of 2M LiF.
(ln 2 = 0.693, R = 8.314 J K^{-1} mol^{-1} and F = 96500 C mol^{-1})

(a) 5.90 V (b) 5.937 V
(c) 5.88 V (d) 4.9 V
(e) 4.8 V

115. Consider the galvanic cell, $Pt(s) | H_2(1 \text{ bar}) | HCl\ (aq)\ (1M) | Cl_2(1 \text{ bar}) | Pt(s)$. After running the cell for sometime, the concentration of the electrolyte is automatically raised to 3M HCl. Molar conductivity of the 3M HCl is about 240 S cm^2 mol^{-1} and limiting molar conductivity of HCl is about 420 S cm^2 mol^{-1}. If K_b of water is 0.52 K kg mol^{-1}, calculate the boiling point of the electrolyte at the end of the experiment

(a) 375.6 K (b) 376.3 K
(c) 378.1 K (d) 380.3 K
(e) 381.6 K

116. The data given below are for the reaction of A and D_2 to form product at 295 K. Find the correct rate expression for this reaction

D_2/mol L^{-1}	A/mol L^{-1}	Initial rate/mol $L^{-1}s^{-1}$
0.05	0.05	$1 \lambda 10^{-3}$
0.15	0.05	$3 \lambda 10^{-3}$
0.05	0.15	$9 \lambda 10^{-3}$

(a) $k[D_2]^1[A]^2$ (b) $k[D_2]^2[A]^1$
(c) $k[D_2]^1[A]^1$ (d) $k[D_2]^2[A]^2$
(e) $k[D_2]^1[A]^0$

117. Find the unit of the rate constant of a reaction represented with a rate equation, rate $= k[A]^{1/2}[B]^{3/2}$

(a) $mol^{-1}L\ s^{-1}$
(b) s^{-1}
(c) mol L^{-1} s^{-1}
(d) mol^{-2} L^2 s^{-1}
(e) mol^{-3} L^3 s^{-1}

118. Under what condition the order of the reaction,
$2HI \longrightarrow H_2(g) + I_2(g)$, is zero

(a) at high temperature
(b) at high partial pressure of HI
(c) at low partial pressure of HI
(d) at high partial pressure of H_2
(e) at high partial pressure of I_2

119. Which of the following statement is true about the adsorption?

(a) $\Leftarrow H < 0$ and $\Leftarrow S < 0$
(b) $\Leftarrow H > 0$ and $\Leftarrow S < 0$
(c) $\Leftarrow H < 0$ and $\Leftarrow S > 0$
(d) $\Leftarrow H = 0$ and $\Leftarrow S < 0$
(e) $\Leftarrow H = 0$ and $\Leftarrow S > 0$

120. In NH_3 synthesis by Haber's process, what is the effect on the rate of the reaction with the addition of Mo and CO, respectively?

(a) Increases and decreases
(b) Decreases and decreases
(c) Decreases and increases
(d) Both Mo and CO increases the rate
(e) Both Mo and CO does not affect the rate

Mathematics

1. The value of $\dfrac{2(\cos 75° + i\sin 75°)}{0.2(\cos 30° + i\sin 30°)}$ is

(a) $\frac{5}{\sqrt{2}}(1+i)$ (b) $\frac{10}{\sqrt{2}}(1+i)$

(c) $\frac{10}{\sqrt{2}}(1-i)$ (d) $\frac{5}{\sqrt{2}}(1-i)$

(e) $\frac{1}{\sqrt{2}}(1+i)$

2. If the conjugate of a complex number z is $\dfrac{1}{i-1}$, then z is

(a) $\frac{1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i-1}$ (d) $\frac{-1}{i+1}$

(e) $\frac{1}{i}$

3. The value of $\left(i^{18} + \left(\frac{1}{i}\right)^{25}\right)^3$ is equal to

(a) $\frac{1+i}{2}$ (b) $2+2i$ (c) $\frac{1-i}{2}$ (d) $\sqrt{2}-\sqrt{2}i$

(e) $2-2i$

4. The modulus of $\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i}$ is

(a) 2 (b) $\sqrt{2}$

(c) 4 (d) 8

(e) 10

5. If $z = e^{i4\pi/3}$, then $(z^{192} + z^{194})^3$ is equal to

(a) -2 (b) -1 (c) $-i$ (d) $-2i$

(e) 0

6. If a and b are real numbers and $(a+ib)^{11} = 1+3i$, then $(b+ia)^{11}$ is equal to

(a) $i+3$ (b) $1+3i$

(c) $1-3i$ (d) 0

(e) $-i-3$

7. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is

(a) $3x^2 - 19x - 3 = 0$ (b) $3x^2 + 19x - 3 = 0$

(c) $x^2 + 19x + 3 = 0$ (d) $3x^2 - 19x - 19 = 0$

(e) $3x^2 - 19x + 3 = 0$

8. The focus of the parabola $y^2 - 4y - x + 3 = 0$ is

(a) $\left(\frac{3}{4}, 2\right)$ (b) $\left(\frac{3}{4}, -2\right)$ (c) $\left(2, \frac{3}{4}\right)$ (d) $\left(\frac{-3}{4}, 2\right)$

(e) $\left(2, \frac{-3}{4}\right)$

9. If $f: R \to (0, \infty)$ is an increasing function and if $\lim_{x \to 2018} \dfrac{f(3x)}{f(x)} = 1$, then $\lim_{x \to 2018} \dfrac{f(2x)}{f(x)}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

(e) 1

10. If f is differentiable at $x = 1$, then $\lim_{x \to 1} \dfrac{x^2 f(1) - f(x)}{x-1}$ is

(a) $-f'(1)$ (b) $f(1) - f'(1)$

(c) $2f(1) - f'(1)$ (d) $2f(1) + f'(1)$

(e) $f(1) + f'(1)$

11. Eccentricity of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{8}$

(e) $\frac{\sqrt{3}}{16}$

12. The focus of the parabola $(y+1)^2 = -8(x+2)$ is

(a) $(-4, -1)$ (b) $(-1, -4)$ (c) $(1, 4)$ (d) $(4, 1)$

(e) $(-1, 4)$

13. Which of the following is the equation of a hyperbola?

(a) $x^2 - 4x + 16y + 17 = 0$

(b) $4x^2 + 4y^2 - 16x + 4y - 60 = 0$

(c) $x^2 + 2y^2 + 4x + 2y - 27 = 0$

(d) $x^2 - y^2 + 3x - 2y - 43 = 0$

(e) $x^2 + 4x + 6y - 2 = 0$

14. Let $f(x) = px^2 + qx + r$, where p, q, r are constants and $p \neq 0$. If $f(5) = -3f(2)$ and $f(-4) = 0$, then the other root of f is

(a) 3 (b) -7 (c) -2 (d) 2

(e) 6

15. Let $f: \bullet \to \bullet$ satisfy $f(x)f(y) = f(xy)$ for all real numbers x and y. If $f(2) = 4$, then $f\left(\frac{1}{2}\right) =$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1
(e) 2

16. Sum of last 30 coefficients in the binomial expansion of $(1 + x)^{59}$ is

(a) 2^{29} (b) 2^{59}
(c) 2^{58} (d) $2^{59} - 2^{29}$
(e) 2^{60}

17. $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 =$

(a) $20\sqrt{6}$ (b) $30\sqrt{6}$ (c) $5\sqrt{10}$ (d) $40\sqrt{6}$
(e) $10\sqrt{6}$

18. Three players A, B and C play a game. The probability that A, B and C will finish the game are respectively $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The probability that the game is finished is

(a) $\frac{1}{8}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{3}{4}$
(e) $\frac{1}{2}$

19. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$ is

(a) 2 (b) $2\sqrt{2}$ (c) 3 (d) $\sqrt{3}$
(e) 1

20. If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \to \infty} f(x)$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0
(e) ∞

21. The value of $\sin \frac{31}{3}\pi$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{-1}{\sqrt{2}}$
(e) $\frac{1}{2}$

22. The sum of odd integers from 1 to 2001 is

(a) $(1121)^2$ (b) $(1101)^2$
(c) $(1001)^2$ (d) $(1021)^2$
(e) $(1011)^2$

23. If $y = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$, then $y'(x)$ is equal to

(a) $2\cos^2 x$ (b) $2\cos^3 x$ (c) $-\cos 2x$ (d) $\cos 2x$
(e) $3\cos x$

24. The foci of the hyperbola $16x^2 - 9y^2 - 64x + 18y - 90 = 0$ are

(a) $\left(\frac{24 \pm 5\sqrt{145}}{12}, 1\right)$ (b) $\left(\frac{21 \pm 5\sqrt{145}}{12}, 1\right)$
(c) $\left(1, \frac{24 \pm 5\sqrt{145}}{2}\right)$ (d) $\left(1, \frac{21 \pm 5\sqrt{145}}{2}\right)$
(e) $\left(\frac{21 \pm 5\sqrt{145}}{2}, -1\right)$

25. If the sum of the coefficients in the expansions of $(a^2x^2 - 2ax + 1)^{51}$ is zero, then a is equal to

(a) 0 (b) 1 (c) -1 (d) -2
(e) 2

26. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 3.23 (c) 2.57 (d) 3.57
(e) 1.03

27. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $(X = 1)$?

(a) $\frac{1}{2^8}$ (b) $\frac{1}{2^{12}}$ (c) $\frac{1}{2^6}$ (d) $\frac{1}{2^4}$
(e) $\frac{1}{2^5}$

28. The number of diagonals of a polygon with 15 sides is

(a) 90 (b) 45 (c) 60 (d) 70
(e) 10

29. In a class, 40% of students study Maths and Science and 60% of students study Maths. What is the probability of a students studying Science given the student is already studying Maths?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{5}$
(e) $\frac{1}{4}$

30. The eccentricity of the conic $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$
(e) 1

31. If the mean of a set of observations $x_1, x_2, \ldots x_{10}$ is 20, then the mean of $x_1 + 4, x_2 + 8, x_3 + 12, \ldots x_{10} + 40$ is

(a) 34 (b) 32 (c) 42 (d) 38
(e) 40

32. A letter is taken at random from the word "STATISTICS" and another letter is taken at random from the word "ASSISTANT". The probability that they are same letters is

(a) $\frac{1}{45}$ (b) $\frac{13}{90}$ (c) $\frac{19}{90}$ (d) $\frac{5}{18}$
(e) $\frac{9}{10}$

33. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then

(a) $a^2 - b^2 + 2ac = 0$ (b) $(a - c)^2 = b^2 + c^2$
(c) $a^2 + b^2 - 2ac = 0$ (d) $a^2 + b^2 + 2ac = 0$
(e) $a + b + c = 0$

34. If the sides of a triangle are 4, 5 and 6 cm. Then the area (in sq cm) of triangle is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}\sqrt{7}$ (c) $\frac{4}{15}$ (d) $\frac{4}{15}\sqrt{7}$
(e) $\frac{15}{4}\sqrt{7}$

35. In a group of 6 boys and 4 girls, a team consisting of four children is formed such that the team has atleast one boy. The number of ways of forming a team like this is

(a) 159 (b) 209 (c) 200 (d) 240
(e) 212

36. A password is set with 3 distinct letters from the word LOGARITHMS. How many such passwords can be formed?

(a) 90 (b) 720 (c) 80 (d) 72
(e) 120

37. If 5^{97} is divided by 52, the remainder obtained is

(a) 3 (b) 5 (c) 4 (d) 0
(e) 1

38. A quadratic equation $ax^2 + bx + c = 0$, with distinct coefficients is formed. It a, b, c are chosen from the numbers 2, 3, 5, then the probability that the equation has real roots is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
(e) $\frac{2}{3}$

39. $\lim_{x \to \infty} \frac{3x^3 + 2x^2 - 7x + 9}{4x^3 + 9x - 2}$ is equal to

(a) $\frac{2}{9}$ (b) $\frac{1}{2}$ (c) $\frac{-9}{2}$ (d) $\frac{3}{4}$
(e) $\frac{9}{2}$

40. The minimum value of $f(x) = \max\{x, 1 + x, 2 - x\}$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) 0
(e) 2

41. The equations of the asymptotes of the hyperbola $xy + 3x - 2y - 10 = 0$ are

(a) $x = -2, y = -3$ (b) $x = 2, y = -3$
(c) $x = 2, y = 3$ (d) $x = 4, y = 3$
(e) $x = 3, y = 4$

42. If $f(x) = x^6 + 6^x$, then $f'(x)$ is equal to

(a) $12x$ (b) $x + 4$
(c) $6x^5 + 6^x \log(6)$ (d) $6x^5 + x6^{x-1}$
(e) x^6

43. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\frac{\sqrt{52}}{7}$ (b) $\frac{52}{7}$ (c) $\frac{\sqrt{53}}{7}$ (d) $\frac{53}{7}$
(e) 6

44. $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} =$

(a) $\frac{m^2}{n^2}$ (b) $\frac{n^2}{m^2}$
(c) ∞ (d) $-\infty$
(e) 0

45. $\lim_{x \to 0} \frac{(\sqrt{1 + 2x}) - 1}{x} =$

(a) 0 (b) –1 (c) $\frac{1}{2}$ (d) 1
(e) $\frac{-1}{2}$

46. Let f and g be differentiable functions such that $f(3) = 5, g(3) = 7, f'(3) = 13, g'(3) = 6, f'(7) = 2$ and $g'(7) = 0$. If $h(x) = (fog)(x)$, then $h'(3) =$

(a) 14 (b) 12 (c) 16 (d) 0
(e) 10

47. $\dfrac{\sqrt{3}}{\sin(20°)} - \dfrac{1}{\cos(20°)} =$

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) 4
(e) 0

48. A poison variate X satisfies $P(X=1)=P(X=2)$. $P(X=6)$ is equal to

(a) $\frac{4}{45}e^{-2}$ (b) $\frac{1}{45}e^{-1}$
(c) $\frac{1}{9}e^{-2}$ (d) $\frac{1}{4}e^{-2}$
(e) $\frac{1}{45}e^{-2}$

49. Let a and b be 2 consecutive integers selected from the first 20 natural numbers. The probability that $\sqrt{a^2+b^2+a^2b^2}$ is an odd positive integer is

(a) $\frac{9}{19}$ (b) $\frac{10}{19}$
(c) $\frac{13}{19}$ (d) 1
(e) 0

50. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle. A point is chosen inside the circle at random. The probability that the point lies outside the ellipse is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$
(e) $\frac{1}{27}$

51. If the vectors $4\hat{i}+11\hat{j}+m\hat{k}, 7\hat{i}+2\hat{j}+6\hat{k}$ and $\hat{i}+5\hat{j}+4\hat{k}$ are coplanar, then m is equal to

(a) 38 (b) 0 (c) 10 (d) –10
(e) 25

52. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{c}=7\hat{i}+9\hat{j}+11\hat{k}$. Then, the area of the parallelogram with diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is

(a) $4\sqrt{6}$ (b) $\frac{1}{2}\sqrt{21}$
(c) $\frac{\sqrt{6}}{2}$ (d) $\sqrt{6}$
(e) $\frac{1}{\sqrt{6}}$

53. If $|\vec{a}|=3, |\vec{b}|=1, |\vec{c}|=4$ and $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to

(a) 13 (b) 26
(c) –29 (d) –13
(e) –26

54. If $|\vec{a}-\vec{b}|=|\vec{a}|=|\vec{b}|=1$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0
(e) π

55. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}, \vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+9\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $\lambda+\mu$ is equal to

(a) 5 (b) –9
(c) –1 (d) 0
(e) –5

56. The solution of $x^{2/5}+3x^{1/5}-4=0$ are

(a) 1, 1024 (b) –1, 1024
(c) 1, 1031 (d) –1024, 1
(e) –1, 1031

57. If the equations $x^2+ax+1=0$ and $x^2-x-a=0$ have a real common root b, then the value of b is equal to

(a) 0 (b) 1 (c) –1 (d) 2
(e) 3

58. If $\sin\theta-\cos\theta=1$, then the value of $\sin^3\theta-\cos^3\theta$ is equal to

(a) 1 (b) –1 (c) 0 (d) 2
(e) –2

59. Two dice of different colours are thrown at a time. The probability that the sum is either 7 or 11 is

(a) $\frac{7}{36}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{5}{9}$
(e) $\frac{6}{7}$

60. $\frac{1}{9!}+\frac{1}{3!7!}+\frac{1}{5!5!}+\frac{1}{7!3!}+\frac{1}{9!}$ is equal to

(a) $\frac{2^9}{10!}$ (b) $\frac{2^{10}}{8!}$ (c) $\frac{2^{11}}{9!}$ (d) $\frac{2^{10}}{7!}$
(e) $\frac{2^8}{9!}$

61. The order and degree of the differential equation $(y''')^2 + (y'')^3 - (y')^4 + y^5 = 0$ is

(a) 3 and 2 (b) 1 and 2
(c) 2 and 3 (d) 1 and 4
(e) 3 and 5

62. $\int_{-2}^{2} |x|\,dx$ is equal to

(a) 0 (b) 1
(c) 2 (d) 4
(e) $\frac{1}{2}$

63. $\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$ is equal to

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{-\pi}{4}$ (d) $\frac{\pi}{2}$
(e) $\frac{-\pi}{2}$

64. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} (3 - f(x))dx = 7$, then $\int_{-1}^{2} f(x)dx$ is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

65. $\int \frac{xe^x}{(1+x)^2} dx =$

(a) $\frac{e^x}{1+x} + C$ (b) $\frac{e^x}{1+e^x} + C$
(c) $\frac{e^{2x}}{1+e^x} + C$ (d) $\frac{e^{-x}}{1+x} + C$
(e) $\frac{e^{-2x}}{1+x} + C$

66. The remainder when 2^{2000} is divided by 17 is

(a) 1 (b) 2 (c) 8 (d) 12
(e) 4

67. The coefficient of x^5 in the expansion of $(x+3)^8$ is

(a) 1542 (b) 1512
(c) 2512 (d) 12
(e) 4

68. The maximum value of 5 $\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is

(a) 5 (b) 11
(c) 10 (d) –1
(e) 2

69. The area of the triangle in the complex plane formed by z, iz and $z + iz$ is

(a) $|z|$ (b) $|\bar{z}|^2$
(c) $\frac{1}{2}|z|^2$ (d) $\frac{1}{2}|z + iz|^2$
(e) $|z + iz|$

70. Let $f : f(-x) \to f(x)$ be a differentiable function. If f is even, then $f'(0)$ is equal to

(a) 1 (b) 2
(c) 0 (d) –1
(e) $\frac{1}{2}$

71. The coordinate of the point dividing internally the line joining the points $(4, -2)$ and $(8, 6)$ in the ratio $7:5$ is

(a) (16, 18) (b) (18, 16)
(c) $\left(\frac{19}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{8}{3}, \frac{19}{3}\right)$
(e) (7, 3)

72. The area of the triangle formed by the points $(a, b + c)$, $(b, c + a)$, $(c, a + b)$ is

(a) abc (b) $a^2 + b^2 + c^2$
(c) $ab + bc + ca$ (d) 0
(e) $a(ab + bc + ca)$

73. If (x, y) is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$, then

(a) $ax + by = 0$ (b) $ax - by = 0$
(c) $bx + ay = 0$ (d) $bx - ay = 0$
(e) $x = y$

74. The equation of the line passing through (a, b) and parallel to the line $\frac{x}{a} + \frac{y}{b} = 1$ is

(a) $\frac{x}{a} + \frac{y}{b} = 3$ (b) $\frac{x}{a} + \frac{y}{b} = 2$
(c) $\frac{x}{a} + \frac{y}{b} = 0$ (d) $\frac{x}{a} + \frac{y}{b} + 2 = 0$
(e) $\frac{x}{a} + \frac{y}{b} = 4$

75. If the points $(2a, a)$, $(a, 2a)$ and (a, a) enclose a triangle of area 18 sq units, then the centroid of the triangle is equal to

(a) (4, 4) (b) (8, 8)
(c) (–4, – 4) (d) $(4\sqrt{2}, 4\sqrt{2})$
(e) (6, 6)

76. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, −2). The third vertex lies on $y = x + 3$. The cordinates of the third vertex can be

(a) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{-3}{2}\right)$
(c) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (d) $\left(\frac{-1}{4}, \frac{1}{2}\right)$
(e) $\left(\frac{3}{2}, \frac{3}{2}\right)$

77. If $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of straight lines, then $f^2 + g^2$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4
(e) 3

78. If θ is the angle between the pair of straight lines $x^2 - 5xy + 4y^2 + 3x - 4 = 0$, then $\tan^2\theta$ is equal to

(a) $\frac{9}{16}$ (b) $\frac{16}{25}$ (c) $\frac{9}{25}$ (d) $\frac{21}{25}$
(e) $\frac{25}{9}$

79. If $3\hat{i} + 2\hat{j} - 5\hat{k} = x(2\hat{i} - \hat{j} + \hat{k}) + y(\hat{i} + 3\hat{j} - 2\hat{k}) + z(-2\hat{i} + \hat{j} - 3\hat{k})$, then

(a) $x = 1, y = 2, z = 3$ (b) $x = 2, y = 3, z = 1$
(c) $x = 3, y = 1, z = 2$ (d) $x = 1, y = 3, z = 2$
(e) $x = 2, y = 2, z = 3$

80. $\sin 15° =$

(a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ (c) $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1 + \sqrt{3}}{\sqrt{2}}$
(e) $\frac{-(1 + \sqrt{3})}{2\sqrt{2}}$

81. If $\vec{a}$ and $\vec{b} = 3\hat{i} + 6\hat{j} + 6\hat{k}$ are collinear and $\vec{a} \cdot \vec{b} = 27$, then $\vec{a}$ is equal to

(a) $3(\hat{i} + \hat{j} + \hat{k})$ (b) $\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $2\hat{i} + 2\hat{j} + 2\hat{k}$ (d) $\hat{i} + 3\hat{j} + 3\hat{k}$
(e) $\hat{i} - 3\hat{j} + 2\hat{k}$

82. If $|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 30$, then $|\vec{a} \times \vec{b}|$ is equal to

(a) 30 (b) $\frac{30}{25}\sqrt{233}$ (c) $\frac{30}{33}\sqrt{193}$ (d) $\frac{65}{23}\sqrt{493}$
(e) $\frac{65}{13}\sqrt{133}$

83. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then r is equal to

(a) 69 (b) 41 (c) 51 (d) 61
(e) 49

84. Distance between two parallel lines $y = 2x + 4$ and $y = 2x - 1$ is

(a) 5 (b) $5\sqrt{5}$
(c) $\sqrt{5}$ (d) $\frac{1}{5}$
(e) $\frac{3}{\sqrt{5}}$

85. $({}^7C_0 + {}^7C_1) + ({}^7C_2 + {}^7C_3) + \ldots + ({}^7C_6 + {}^7C_7) =$

(a) $2^8 - 2$ (b) $2^7 - 1$
(c) 2^7 (d) $2^8 - 1$
(e) $2^7 - 2$

86. The coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$ is

(a) 17 (b) 19
(c) −17 (d) −19
(e) 20

87. The equation of the circle with centre (2, 2) which passes through (4, 5) is

(a) $x^2 + y^2 - 4x + 4y - 77 = 0$
(b) $x^2 + y^2 - 4x - 4y - 5 = 0$
(c) $x^2 + y^2 + 2x + 2y - 59 = 0$
(d) $x^2 + y^2 - 2x - 2y - 23 = 0$
(e) $x^2 + y^2 + 4x - 2y - 26 = 0$

88. The point in the xy-plane which is equidistant from (2, 0, 3), (0, 3, 2) and (0, 0, 1) is

(a) (1, 2, 3) (b) (−3, 2, 0)
(c) (3, −2, 0) (d) (3, 2, 0)
(e) (3, 2, 1)

89. Let $f : \square \to \square$ be such that $f(1) = 2$ and $f(x + y) = f(x)f(y)$ for all natural numbers x and y. If $\Sigma_{k=1}^{n} f(a + k) = 16(2^n - 1)$, then a is equal to

(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

90. If ${}_nC_{r-1} = 36$, ${}_nC_r = 84$ and ${}_nC_{r+1} = 126$, then $n =$

(a) 3 (b) 4 (c) 8 (d) 9
(e) 10

91. Let $f:(-1,1)\to(-1,1)$ be continuous, $f(x)=f(x^2)$ for all $x\in(-1,1)$ and $f(0)=\frac{1}{2}$, then the value of $4f\left(\frac{1}{4}\right)$ is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

92. $\lim_{x\to\infty}\sqrt{x^2+1}-\sqrt{x^2-1}=$

(a) -1 (b) 1 (c) 0 (d) 2
(e) 4

93. If f is differentiable at $x=1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5$, $f'(1)=$

(a) 0 (b) 1 (c) 3 (d) 4
(e) 5

94. The maximum value of the function $2x^3-15x^2+36x+4$ is attained at

(a) 0 (b) 3 (c) 4 (d) 2
(e) 5

95. If $\int f(x)\cos x\,dx=\frac{1}{2}\{f(x)\}^2+C$, then $f\left(\frac{\pi}{2}\right)$ is

(a) C (b) $\frac{\pi}{2}+C$ (c) $C+1$ (d) $2\pi+C$
(e) $C+2$

96. $\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx=$

(a) $\pi(\sqrt{2}-1)$ (b) $\pi(\sqrt{2}+1)$
(c) $2\pi(\sqrt{2}-1)$ (d) $2\pi(\sqrt{2}+1)$
(e) $\frac{\pi}{\sqrt{2}+1}$

97. $\int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx=$

(a) 2 (b) π (c) $\frac{\pi}{4}$ (d) 2π
(e) 0

98. $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^2}\right)=$

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) 0
(e) $\frac{1}{6}$

99. The area bounded by $y=\sin^2 x$, $x=\frac{\pi}{2}$ and $x=\pi$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{16}$
(e) 2π

100. The differential equation of the family of curves $y=e^x(A\cos x+B\sin x)$, where A and B are arbitrary constants is

(a) $y''-2y'+2y=0$ (b) $y''+2y'-2y=0$
(c) $y''+y'^2+y=0$ (d) $y''+2y'-y=0$
(e) $y''-2y'-2y=0$

101. The real part of $(i-\sqrt{3})^{13}$ is

(a) 2^{-10} (b) 2^{12} (c) 2^{-12} (d) -2^{-12}
(e) 2^{10}

102. $\lim_{x\to 0}\frac{1+x-e^x}{x^2}=$

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$
(c) 1 (d) -1
(e) 0

103. $\int\frac{(\sin x+\cos x)(2-\sin 2x)}{\sin^2 2x}dx=$

(a) $\frac{\sin x+\cos x}{\sin 2x}+C$ (b) $\frac{\sin x-\cos x}{\sin 2x}+C$
(c) $\frac{\sin x}{\sin x+\cos x}+C$ (d) $\frac{\sin x}{\sin x-\cos x}+C$
(e) $\frac{\sin x-\cos x}{\sin x+\cos x}+C$

104. A plane is at a distance of 5 units from the origin and perpendicular to the vector $2\hat{i}+\hat{j}+2\hat{k}$. The equation of the plane is

(a) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=15$ (b) $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=15$
(c) $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})=15$ (d) $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=15$
(e) $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=15$

105. $\frac{\sin A-\sin B}{\cos A+\cos B}$ is equal to

(a) $\sin\left(\frac{A+B}{2}\right)$ (b) $2\tan(A+B)$
(c) $\cot\left(\frac{A-B}{2}\right)$ (d) $\tan\left(\frac{A-B}{2}\right)$
(e) $2\cot(A+B)$

106. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2} =$

(a) x (b) $-16x$ (c) $15x$ (d) $16x$
(e) $-15x$

107. The arithmetic mean of ${}^nC_o, {}^nC_1, {}^nC_2 \dots, {}^nC_n$ is

(a) $\frac{2^n}{n+1}$ (b) $\frac{2^n}{n}$ (c) $\frac{2^{n-1}}{n+1}$ (d) $\frac{2^{n-1}}{n}$
(e) $\frac{2^{n+1}}{n}$

108. The variance of first 20 natural numbers is

(a) $\frac{399}{2}$ (b) $\frac{379}{12}$ (c) $\frac{133}{2}$ (d) $\frac{133}{4}$
(e) $\frac{169}{2}$

109. If S is a set with 10 elements and $A = \{(x, y) : x, y \in S, x \neq y\}$, then the number of elements in A is

(a) 100 (b) 90 (c) 80 (d) 150
(e) 45

110. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows 3 is

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{11}{12}$
(e) $\frac{1}{11}$

111. If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$, then the sum of all the diagonal entries of A^{-1} is

(a) 2 (b) 3 (c) -3 (d) -4
(e) 4

112. Let $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$. If $x = -9$ is a root of $f(x) = 0$, then the other roots are

(a) 2 and 7 (b) 3 and 6 (c) 7 and 3 (d) 6 and 2
(e) 6 and 7

113. If $[1 \quad x \quad 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$, then x can be

(a) -2 (b) 2 (c) 14 (d) -14
(e) 0

114. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then $x =$

(a) 2 (b) $\frac{1}{2}$
(c) 1 (d) 3
(e) 0

115. If $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then $5a + 4b + 3c + 2d + e$ is equal to

(a) 11 (b) -11 (c) 12 (d) -12
(e) 13

116. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} =$

(a) 1 (b) 0
(c) $(1-a)(1-b)(1-c)$ (d) $a+b+c$
(e) $2(a+b+c)$

117. If $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$, then $f(50) =$

(a) 0 (b) 2 (c) 4 (d) 1
(e) 3

118. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)\, dx =$

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) -1
(e) 0

119. The equation of the plane passing through the points $(1, 2, 3)$, $(-1, 4, 2)$ and $(3, 1, 1)$ is

(a) $5x + y + 12z = 23$ (b) $5x + 6y + 2z = 23$
(c) $5x - 6y + 2z = 23$ (d) $x + y + z = 13$
(e) $2x + 6y + 5z = 7$

120. In an arithmetic progression, if the kth term is $5k + 1$, then the sum of first 100 terms is

(a) 50(507) (b) 51(506)
(c) 50(506) (d) 51(507)
(e) 52(506)

Answers

Physics & Chemistry

1.	*(b)*	2.	*(c)*	3.	*(b)*	4.	*(d)*	5.	*(c)*	6.	*(e)*	7.	*(a)*	8.	*(d)*	9.	*(a)*	10.	*(b)*
11.	*(c)*	12.	*(a)*	13.	*(e)*	14.	*(a)*	15.	*(b)*	16.	*(b)*	17.	*(b)*	18.	*(b)*	19.	*(d)*	20.	*(a)*
21.	*(b)*	22.	*(c)*	23.	*(d)*	24.	*(a)*	25.	*(d)*	26.	*(c)*	27.	*(b)*	28.	*(a)*	29.	*(d)*	30.	*(e)*
31.	*(e)*	32.	*(e)*	33.	*(d)*	34.	*(a)*	35.	*(c)*	36.	*(e)*	37.	*(a)*	38.	*(a)*	39.	*(b)*	40.	*(b)*
41.	*(a)*	42.	*(e)*	43.	*(c)*	44.	*(e)*	45.	*(d)*	46.	*(d)*	47.	*(b)*	48.	*(c)*	49.	*(c)*	50.	*(a)*
51.	*(b)*	52.	*(c)*	53.	*(d)*	54.	*(c)*	55.	*(d)*	56.	*(a)*	57.	*(b)*	58.	*(d)*	59.	*(a)*	60.	*(b)*
61.	*(c)*	62.	*(e)*	63.	*(a)*	64.	*(d)*	65.	*(d)*	66.	*(d)*	67.	*(e)*	68.	*(a)*	69.	*(d)*	70.	*(e)*
71.	*(e)*	72.	*(c)*	73.	*(c)*	74.	*(d)*	75.	*(c)*	76.	*(d)*	77.	*(c)*	78.	*(d)*	79.	*(b)*	80.	*(e)*
81.	*(b)*	82.	*(a)*	83.	*(a)*	84.	*(c)*	85.	*(a)*	86.	*(b)*	87.	*(e)*	88.	*(e)*	89.	*(a)*	90.	*(c)*
91.	*(a)*	92.	*(a)*	93.	*(c)*	94.	*(c)*	95.	*(c)*	96.	*(d)*	97.	*(c)*	98.	*(e)*	99.	*(d)*	100.	*(a)*
101.	*(e)*	102.	*(d)*	103.	*(a)*	104.	*(a)*	105.	*(a)*	106.	*(a)*	107.	*(b)*	108.	*(e)*	109.	*(b)*	110.	*(a)*
111.	*(b)*	112.	*(b)*	113.	*(a)*	114.	*(a)*	115.	*(a)*	116.	*(a)*	117.	*(a)*	118.	*(b)*	119.	*(a)*	120.	*(a)*

Mathematics

1.	*(b)*	2.	*(d)*	3.	*(e)*	4.	*(a)*	5.	*(b)*	6.	*(e)*	7.	*(e)*	8.	*(d)*	9.	*(e)*	10.	*(c)*
11.	*(a)*	12.	*(a)*	13.	*(d)*	14.	*(a)*	15.	*(b)*	16.	*(c)*	17.	*(d)*	18.	*(d)*	19.	*(b)*	20.	*(a)*
21.	*(a)*	22.	*(c)*	23.	*(c)*	24.	*(a)*	25.	*(b)*	26.	*(c)*	27.	*(b)*	28.	*(a)*	29.	*(c)*	30.	*(b)*
31.	*(c)*	32.	*(c)*	33.	*(a)*	34.	*(e)*	35.	*(b)*	36.	*(b)*	37.	*(b)*	38.	*(a)*	39.	*(d)*	40.	*(b)*
41.	*(b)*	42.	*(c)*	43.	*(a)*	44.	*(a)*	45.	*(d)*	46.	*(b)*	47.	*(d)*	48.	*(a)*	49.	*(d)*	50.	*(b)*
51.	*(c)*	52.	*(a)*	53.	*(d)*	54.	*(a)*	55.	*(b)*	56.	*(d)*	57.	*(c)*	58.	*(a)*	59.	*(b)*	60.	*(a)*
61.	*(a)*	62.	*(d)*	63.	*(b)*	64.	*(e)*	65.	*(a)*	66.	*(a)*	67.	*(b)*	68.	*(c)*	69.	*(c)*	70.	*(c)*
71.	*(c)*	72.	*(d)*	73.	*(d)*	74.	*(b)*	75.	*(b)*	76.	*(c)*	77.	*(b)*	78.	*(c)*	79.	*(c)*	80.	*(a)*
81.	*(b)*	82.	*(e)*	83.	*(b)*	84.	*(c)*	85.	*(c)*	86.	*(d)*	87.	*(b)*	88.	*(d)*	89.	*(a)*	90.	*(d)*
91.	*(b)*	92.	*(c)*	93.	*(e)*	94.	*(d)*	95.	*(c)*	96.	*(a,e)*	97.	*(c)*	98.	*(d)*	99.	*(b)*	100.	*(a)*
101.	*(*)*	102.	*(b)*	103.	*(b)*	104.	*(c)*	105.	*(d)*	106.	*(b)*	107.	*(a)*	108.	*(d)*	109.	*(b)*	110.	*(b)*
111.	*(e)*	112.	*(a)*	113.	*(a,d)*	114.	*(b)*	115.	*(b)*	116.	*(b)*	117.	*(a)*	118.	*(a)*	119.	*(b)*	120.	*(a)*

Note *(*) None of the option is correct.*

Answer *with* Explanations

Physics

1. *(b)* Given, figure shows the relation between speed and time. So, area of the figure will be displacement.

Therefore,

area of first triangular $(d_1) = \frac{1}{2} \times 10 \times 10 = 50\text{ m}$,

area of second triangular

$$(d_2) = \frac{1}{2} \times (-10) \times 10 = -50\text{ m}$$

Therefore, total area (d)
= total displacement of particle

$$\Rightarrow \quad d_1 + d_2 = 50 - 50 = 0$$

2. *(c)* Given,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or $$T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore \quad 2\frac{\Delta T}{T} = \frac{\Delta l}{l} + \frac{\Delta g}{g} \quad \ldots\text{(i)}$$

Now, $l = 50\text{ cm}$, $\Delta l = 2\text{ mm} = 0.2\text{ cm}$

$$\frac{\Delta g}{g} = 1.1\% = \frac{1.1}{100}$$

Put these values in Eq. (i), then we get

$$\frac{\Delta T}{T} = \frac{1}{2}\left[\frac{0.2}{50} + \frac{1.1}{100}\right]$$

$$= 7.5 \times 10^{-3}\text{ s or } 7.5\text{ ms}$$

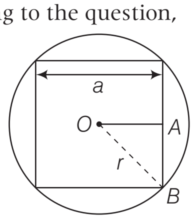
$\because$ In 100 s , resolution of clock is 7.5 ms.

$\therefore$ In 60 s resolution of clock is

$$\frac{7.5 \times 60}{100} \approx 5\text{ ms}$$

Hence, option (c) is correct.

3. *(b)* According to the question,



In the triangular OAB,

$$(OB)^2 = (OA)^2 + (AB)^2$$

or $$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$

$$r = \sqrt{\frac{2a^2}{4}} = \frac{a}{\sqrt{2}}$$

4. *(d)* $\because$ Least count $=$ MSD $-$ VSD

$$\because \quad \text{MSD} = \frac{\left(\frac{30}{100}\right)}{600} = 5 \times 10^{-4}\text{ m}$$

and $$\text{VSD} = \frac{19}{20} \times \text{MSD}$$

$$= \frac{19}{20} \times 5 \times 10^{-4} = 4.75 \times 10^{-4}$$

$\therefore$ Least count $= (5 - 4.75) \times 10^{-4} = 0.025\text{ mm}$

5. *(c)* Given, $V(x) = \frac{kx^2}{2} + \frac{\lambda}{x}$

$\because$ In electrical analogy,

$$\omega = \frac{1}{\sqrt{LC}}$$

but in mechanical analogy L and C

ω will be transformed into m and $\frac{1}{k}$.

Here, m is mass of particle.

Hence, angular frequency, $\omega = \frac{1}{\sqrt{m\left(\frac{1}{k}\right)}}$ or $\omega = \sqrt{\frac{k}{m}}$

6. *(e)* Path of the centre of mass in a two particle system,

$$\mathbf{r}(t) = \left[\frac{m_1 \mathbf{r}_1(t) + m_2 \mathbf{r}_2(t)}{m_1 + m_2}\right]$$

or $$\mathbf{r}(t) = \left[\frac{m(t\,\hat{\mathbf{i}} - t^3\,\hat{\mathbf{j}} + 2t^2\,\hat{\mathbf{k}} + 2m(t\,\hat{\mathbf{i}} - t^3\hat{\mathbf{j}} - t^2\,\hat{\mathbf{k}})}{m + 2m}\right]$$

$$\Rightarrow \quad \mathbf{r}(t) = \frac{3t\,\hat{\mathbf{i}} - 3t^3\hat{\mathbf{j}}}{3} = t\,\hat{\mathbf{i}} - t^3\hat{\mathbf{j}}$$

According above result option (e) is correct.

7. *(a)* Given,

$$\frac{R_A}{R_B} = \frac{3}{1} \quad \ldots\text{(i)}$$

and $$\rho_A = \rho_B \quad \ldots\text{(ii)}$$

$\because$ Average density, $\rho = \dfrac{3g}{4\pi RG}$

$\therefore$ From Eq. (ii),

$$\Rightarrow \quad \frac{3g_A}{4\pi R_A G} = \frac{3g_B}{4\pi R_B G} \quad \Rightarrow \quad \frac{g_A}{g_B} = \frac{R_A}{R_B}$$

From Eq. (i),

$$\frac{g_A}{g_B} = \frac{3}{1}$$

8. *(d)* $\because F = qvB$

$$\therefore \quad B = \frac{F}{qv} = \frac{F}{q\left(\frac{d}{t}\right)} = \frac{F}{Id} = \frac{T}{I}$$

$$\therefore \quad B = \frac{\text{Surface tension}}{\text{Current}}$$

9. *(a)* Einstein was awarded the Nobel Prize for his work in photoelectric effect.

10. *(b)* $\because$ Momentum before particle is attached to the ring, $= I\omega = mR^2\omega$

Momentum after two particle is attached to the ring, $= I\omega/2 + (\mu x^2)\,\omega/2$

Here, $\mu = \dfrac{M \cdot M}{M + M} = \dfrac{M}{2}$ and $x = 2R$

$$\text{So, } I\frac{\omega}{2} + (\mu\, x^2)\frac{\omega}{2} = \left[mR^2 + \frac{M}{2}(4R^2)\right]\frac{\omega}{2}$$

$$= [mR^2 + 2MR^2]\frac{\omega}{2}$$

According law of conservation of momentum,

(momentum before) = (momentum after)

$$\Rightarrow \quad mR^2\omega = (mR^2 + 2MR^2)\frac{\omega}{2}$$

$$\Rightarrow \quad 2mR^2 = mR^2 + 2MR^2$$

$$\text{So} \quad \frac{m}{M} = 2$$

11. *(c)* $\because$ According to the question,

$$\frac{1}{2}mv_f^2 = \frac{1}{2} \times \frac{1}{2}mv_i^2$$

(v_i, v_f = initial and final speeds of the body)

$$\text{or} \quad v_f^2 = \frac{v_i^2}{2} \quad \text{or} \quad v_f = \frac{10}{\sqrt{2}}$$

Given, $\quad f = -kv$

$$\text{or} \quad ma = -kv \Rightarrow \frac{mdv}{dt} = -kv$$

$$\Rightarrow \quad \int_{10}^{10/\sqrt{2}} \frac{1}{v}dv = -\int_0^{10} kdt \qquad (\because m = 1\text{kg})$$

$$\Rightarrow \quad (\ln v)_{10}^{10/\sqrt{2}} = -k(10)$$

$$\Rightarrow \ln\frac{10}{\sqrt{2}} - \ln 10 = -k(10)$$

$$\Rightarrow \quad k = \frac{1}{10}\ln\left(\frac{10}{10/\sqrt{2}}\right) = \frac{1}{10}\ln\sqrt{2} = \frac{\ln 2}{20}$$

12. *(a)* Given that, $\mathbf{r}(t) = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}}$

$$\because \mathbf{v} = \frac{d\mathbf{r}(t)}{dt} = -a\omega\sin\omega t\;\hat{\mathbf{i}} + a\omega\cos\omega t\;\hat{\mathbf{j}}$$

$$\mathbf{a} = \frac{dv}{dt} = -a\omega^2\cos\omega t\,\hat{\mathbf{i}} - a\omega^2\sin\omega t\,\hat{\mathbf{j}}$$

$$\because \mathbf{a}\cdot\mathbf{v} = (-a\omega\sin\omega t\,\hat{\mathbf{i}} + a\omega\cos\omega t\,\hat{\mathbf{j}})$$
$$(-a\omega^2\cos\omega t\;\hat{\mathbf{i}} - a\omega^2\sin\omega t\;\hat{\mathbf{j}})$$

$$\mathbf{a}\cdot\mathbf{v} = a^2\omega^3\sin\omega t\cos\omega t - a^2\omega^3\sin\omega t\cos\omega t$$

$$\Rightarrow \mathbf{a}\cdot\mathbf{v} = 0 \;\; [\because \mathbf{a}\cdot\mathbf{v} = |\mathbf{a}||\mathbf{v}|\cos\theta]$$

Above result implies that acceleration is perpendicular to velocity.

13. *(e)* According work-energy theorem,

net work = change in kinetic energy = final KE – initial KE

$$\text{So} \quad F\cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad ma\cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad v^2 - u^2 = 2as \;\;\Rightarrow\;\; v^2 = u^2 + 2as$$

14. *(a)* $\because$ In option (a), $\dfrac{aT}{v} = \dfrac{\frac{m}{s^2}\times s}{\left(\frac{m}{s}\right)} = \dfrac{m\times s^2}{m\times s^2} = [M^0L^0T^0]$

So, according to the result above relation does not depend on time. So, option (a) is correct.

15. *(b)* $\because$ Young's modulus, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} \qquad \ldots(\text{i})$$

$\because$ Force exerted due to its own weight $= \dfrac{mg}{2}$

$$\therefore \quad Y = \frac{\left(\frac{mg}{2}\right)}{A\left(\frac{0.5}{L}\right)} = \left(\frac{mg}{AL}\right)$$

$$\text{So} \quad L = \frac{Y}{\left(\frac{mg}{AL}\right)} = \frac{Y}{dg} \qquad \left(\because \frac{m}{AL} = d\right)$$

16. *(b)* In a projectile motion, speed (v) of a projectile decreases with time (t).
Hence, graph in option (b) is correct.

17. *(b)* Let, **for body** P, volume $= V_P$

Given, Immersed volume $= \frac{V_P}{2}$

For body Q, volume $= V_Q$

Immersed volume $= \frac{2}{3}V_Q$

According to Archimedes principle,
$\because$ Weight of body = weight of fluid displaced
For body P,

$$V_P \rho_P g = \left(\frac{1}{2}V_P\right)\rho_w g$$

So, $$\frac{\rho_P}{\rho_w} = \frac{1}{2} \quad \ldots(\text{i})$$

For body Q,

$$V_Q \rho_Q g = \left(\frac{2}{3}V_Q\right)\left(\frac{3}{4}\right)\rho_w g$$

$$(\because \text{density of liquid} = \frac{3}{4}\rho_w)$$

So, $$\frac{\rho_Q}{\rho_w} = \frac{1}{2} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii),

$$\frac{\rho_P}{\rho_Q} = 1 \text{ or } \rho_P : \rho_Q = 1 : 1$$

18. *(b)* Given, $v = 320\ \text{ms}^{-1}$, $l = 1\ \text{m}$

For an organ pipe whose one end is closed, only odd harmonics containing odd multiples of fundamental frequecy are present.

Resonate frequency of first mode, $n_1 = \frac{v}{4l}$,

Second mode, $n_2 = 3n_1$,
Third mode, $n_3 = 5n_1$,
Fourth mode, $n_4 = 7n_1$,
Fifth mode, $n_5 = 9n_1$

So, $n_1 = \frac{320}{4} = 80\ \text{Hz}$

$n_2 = 3 \times 80 = 240\ \text{Hz}$
$n_3 = 5 \times 80 = 400\ \text{Hz}$
$n_4 = 7 \times 80 = 560\ \text{Hz}$
$n_5 = 9 \times 80 = 720\ \text{Hz}$

So according result, option (b) as multiple of cannot a resonating frequency of the pipe.

19. *(d)* $\because$ Given,

$$y_1 = \frac{5}{(3x-4t)^2+2},\ y_2 = \frac{-5}{(3x+4t-6)^2+2}$$

According option (d), at $x = 1$

$$y_1 = \frac{5}{(3-4t)^2+2}$$

$$y_2 = \frac{-5}{(3+4t-6)^2+2} = \frac{-5}{(3-4t)^2+2}$$

Both wave pulse equation are existing in same string therefore resultant equation of wave pulse.

$$y = y_1 + y_2 = 0$$

Hence, option (d) is correct.

20. *(a)* $\because$ Wave equation, $y = A_0 \sin(kx - \omega t)$ ...(i)

where, k = angular wave number $= \frac{2\pi}{\lambda}$

A_0 = amplitude

$\because \quad v_{max} = a\omega$

$\therefore \quad \omega = 1$

$$\left(\because \text{given that } v_{max} = 1\frac{m}{s},\ a = 1\frac{m}{s^2}\right)$$

$\because \quad k = 1$ (given)

$\therefore$ From Eq. (i),

$$y = \sin(x - t)$$

21. *(b)* According to question, $\frac{X_A}{X_B} = \frac{K}{1}$ ...(i)

$$\frac{U_A}{U_B} = \frac{1}{K^2}$$

So, $U_A = y$ and $U_B = K^2 y$

According to de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mK_1}} \quad \ldots(\text{ii})$$

Here, K_1 is kinetic energy.
So, from Eqs. (i) and (ii),

$$\Rightarrow \frac{\left(\frac{h}{\sqrt{2mK_{1A}}}\right)}{\left(\frac{h}{\sqrt{2mK_{1B}}}\right)} = \frac{K}{1} \Rightarrow \sqrt{\frac{K_{1B}}{K_{1A}}} = \frac{K}{1}$$

$$\Rightarrow \frac{K_{1B}}{K_{1A}} = K^2$$

So, $K_{1B} = K^2 x$ and $K_{1A} = x$

$\because$ Total energy, $E = K_1 + U$

So, $E_A = K_{1A} + U_A$

and $E_B = K_{1B} + U_B$

or $\frac{E_A}{E_B} = \frac{K_{1A} + U_A}{K_{1B} + U_B} = \frac{x + y}{K^2 x + K^2 y}$

$\Rightarrow$ So, $\frac{E_A}{E_B} = \frac{1}{K^2}$ (or) $E_A : E_B = 1 : K^2$

22. *(c)* Given, $a \propto x(t)$ ...(i)

According option (a),

$$x(t) = \sin \omega t,\ \omega > 0$$

$\therefore$ $u(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\sin \omega t = \omega \cos \omega t$ and a

$$a = \frac{du(t)}{dt} = \frac{d}{dt}(\omega \cos \omega t) = -\omega^2 \sin \omega t$$

or $a = -\omega^2 x(t)$ ...(ii)

According option (b),

$$x(t) = \sin \omega t + \cos \omega t,\ \omega > 0$$

$\therefore$ $v(t) = \frac{dx(t)}{dt} = \omega \cos \omega t - \omega \sin \omega t$

and $a = \frac{dv(t)}{dt} = -\omega^2 \sin \omega t - \omega^2 \cos \omega t$

or $a = -(\omega^2 \sin \omega t + \omega^2 \cos \omega t)$ or $a = -\omega^2 x(t)$ ...(iii)

According option (c),

$$x(t) = e^{\omega t},\ \omega > 0$$

$\therefore$ $v(t) = \frac{dx(t)}{dt} = \omega e^{\omega t}$

and $a = \frac{dv(t)}{dt} = \omega^2 e^{\omega t}$

or $a = \omega^2 x(t)$ ...(iv)

According option (d),

$$x(t) = e^{\omega t} + \sin \omega t\ \ \omega > 0$$

$\therefore$ $v(t) = \frac{dx(t)}{dt} = \omega e^{\omega t} + \omega \cos \omega t$

and $a = \frac{dv(t)}{dt} = \omega^2 e^{\omega t} - \omega^2 \sin \omega t$

or $a = \omega^2 (e^{\omega t} - \sin \omega t)$ ...(v)

According option (e),

$$x(t) = e^{\omega_1 t} + e^{-\omega_2 t},\ \omega_1 \text{ and } \omega_2 > 0$$

$\therefore$ $v(t) = \frac{dx(t)}{dt} = \omega_1 e^{\omega_1 t} - \omega_2 e^{-\omega_2 t}$

and $a = \frac{dv(t)}{dt} = \omega_1^2 e^{\omega_1 t} + \omega_2^2 e^{-\omega_2 t}$

or $a = \omega_1^2\, e^{\omega_1 t} + \omega_2^2\, e^{-\omega_2 t}$ ...(vi)

Hence by comparing Eqs. (ii), (iii), (iv), (v) and (vi) with Eq. (i), option (c) is correct.

23. *(d)* $\because$ Total momentum of block = Area of given graph

$\therefore$ $P = A_1 + A_2 + A_3$

or $P = \frac{1}{2} \times 1 \times 2 - \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 2$

or $P = 0$ or $mv = 0$ or $v = 0$,

Hence, kinetic energy at $(t = 45) = \frac{1}{2}mv^2 = 0$

24. *(a)* $\because$ Speed of transverse wave

$$v = \sqrt{\frac{\text{Stress}}{\text{density of wire}}} \text{ or } v = \sqrt{\frac{\sigma}{d}}$$

According to question, if σ increases 2 times

then $v_1 = \sqrt{\frac{2\sigma}{d}} = \sqrt{2}\sqrt{\frac{\sigma}{d}} = \sqrt{2}v$

Hence, speed of transverse waves along the wire changes by $\sqrt{2}$ times.

25. *(d)* Surface area new bubble = surface area of first bubble + surface area of second bubble

$\Rightarrow$ $A = A_1 + A_1$

$\because$ Surface area of a bubble, $A = 8\pi R^2$

$\therefore$ $8\pi R^2 = 8\pi R_1^2 + 8\pi R_2^2$

$\Rightarrow$ $R^2 = R_1^2 + R_2^2$

(give that $R_1 = 3\text{ mm}$, $R_2 = 4\text{ mm}$)

$\Rightarrow$ $R^2 = (3)^2 + (4)^2$

$\Rightarrow$ $R = 5\text{ mm}$

26. *(c)* Given $L = 0.05\text{ H}$, $C = 80\,\mu\text{F}$

$$V_{\max} = 200\text{ V}$$

$\because$ Voltage equation $V(t) = V_m \sin \omega t$

$\therefore$ Current $(i) = \frac{cdv}{dt} = c\frac{d}{dt}V_m \sin \omega t = CV_m \omega \cos \omega t$

$\because$ $\omega = \frac{1}{LC}$

$\therefore$ $i = V_m \sqrt{\frac{C}{L}} \cos \omega t$

$\therefore$ Maximum current (i_m), $V_m \sqrt{\frac{C}{L}} = 200 \times \sqrt{\frac{80\mu}{0.05}}$

$$i_m = 8\text{ A}$$

27. *(b)* $\because$ Equation of displacement of particle is

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t) \quad \text{...(i)}$$

or $y = 4\left[\left(\frac{1 + \cos t}{2}\right)\sin 1000t\right]$

$$\left(\because \cos^2 x = \frac{1 + \cos 2x}{2}\right)$$

or $y = 3\sin 1000t + 2\cos t \sin 1000t$

or $y = 2\sin 1000t + \sin 1001t + \sin t\ 999t$

$[2\sin A \cos B = \sin(A+B) + \sin(A-B)]$

Hence, Eq. (i) is the result of the superposition of three simple harmonic motion.

28. *(a)* For a cylindrical tube

If both the ends is closed then

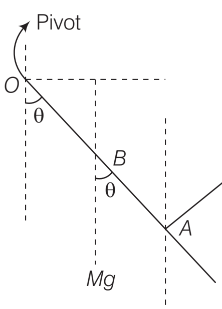
fundamental frequency, $n = \frac{v}{2l}$ ...(i)

If one of the ends is closed then fundamental frequency, $n_1 = \frac{v}{4l}$ ...(ii)

from equation (i) and (ii)

$$n_1 = \frac{n}{2}$$

29. *(d)* Let the length of bar is l, then according to the problem.



For equilibrium of the bar,

$$F_{ext}^{net} = 0 \text{ and } \tau_{ext}^{net} = 0$$

taking torque about pivot O,

$$\mathbf{OB} \times \mathbf{Mg} + \mathbf{OA} \times \mathbf{F} = 0$$

$\Rightarrow \quad -OB\ Mg\sin\theta + OA.F\sin 90° = 0$

(clockwise) (anti-clockwise)

$\therefore \quad lF = \frac{l}{2} Mg\sin\theta$

$\Rightarrow \quad F = \frac{Mg\sin\theta}{2}$

30. *(e)* $\because$ Total energy of particle $= mc^2 - m_oc^2$

rest mass energy of particle $= m_0c^2$

according question,

$\Rightarrow \quad mc^2 - m_0c^2 = 2m_0c^2$

or $\quad m = 3m_0$

When a particle collides with another particle then net mass of new particle will be $m = 3m_0$

31. *(e)* Electrical force between two charge

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

dimension of $\varepsilon_0 = \frac{[A^2T^2]}{[MLT^{-2}][L^2]}$

$= [M^{-1}L^{-3}T^4A^2]$

32. *(e)* $\because$ The displacement of wave

$$y = 0.6 \times 10^{-3}\sin(500t - 0.05x)$$

$\therefore \quad v = \frac{dy}{dt} = 0.6 \times 10^{-3} \times 500\sin(500t - 0.05x)$

$v = 0.3\sin(500t - 0.05x)$

For maximum particle velocity,

$$\sin(500t - 0.05x) = 1$$

Hence, $\quad V_{max} = 0.3\ \text{ms}^{-1}$

33. *(d)* $\because$ According Einstein's photoelectric equation,

$$\frac{1}{2}mv_{max}^2 = hv - w_0 \quad \text{...(i)}$$

Here, w_0 is work function and

v is frequency of photon.

According question equation, electric field repersenting given as, is

$$E = 200\{\sin(4\pi \times 10^{10})t + \sin(4\pi \times 10^{15})t\}$$

Here, fundamental frequency of above equation will be LCM of both component frequency.

Hence, the fundamental frequencyes

$$\omega = 4\pi \times 10^{15}\ \text{rad}$$

$\because \ \omega = 2\pi\nu$ or h, $\nu = \frac{4\pi \times 10^{15}}{2\pi} = 2\pi \times 10^{15}$ Hz

Put the value of h, ν and W_0 in Eq. (i)

$KE_{max} = (6.63 \times 10^{-34} \times 2\pi \times 10^{15} - 2 \times 1.6 \times 10^{19}) + \hat{\mathbf{j}}$,

Hence $KE_{max} = 6.3$ e.V

34. *(a)* $\because$ de-Broglie wavelength λ_n of the electron in the nth orbit of hydrogen atom

$$\lambda = \frac{2\pi}{nh} \times hr$$

$\Rightarrow \quad \lambda_n = \frac{2\pi r}{n}$

$$\left[\begin{array}{l}\text{As, } \lambda = \frac{h}{p} = \frac{h}{mv} \\ \text{or } mvr = \frac{hr}{\lambda} = \frac{nh}{2\pi},\ \lambda = \frac{2\pi}{nh} \times hr\end{array}\right]$$

$$\lambda_n \times \frac{1}{n}$$

Hence, λ_n is inversely proportional to n.

35. *(c)* According first law of thermodynamics,

$$Q = \Delta V + W$$

or $\Delta V = Q - W$

According given conditions

I. $Q > 0$ and $W = 0$

or $\Delta V = Q$

Here $Q > 0$, therefore ΔV will be increase.

II. $Q < 0$ and $W = 0$

or $\Delta V = Q$

Here, $Q > 0$, therefore ΔV will be decrease.

III. $W > 0$ and $Q = 0$

or $\Delta V = -W$

Here, $W > 0$ therefore ΔV will be decrease.

IV. $W < 0$ and $Q = 0$

or $\Delta V = -W$

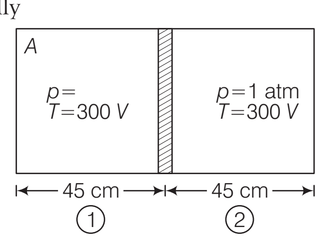
Here, $W < 0$, therefore ΔV will be increase.

Hence, condition I and IV will lead to an increase in the internal energy of the system.

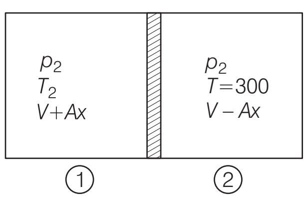
Therefore, option (c) is correct.

36. *(e)* Let the crossectional area of the cylinder is A.

Initially



Representing a closed cylinder separated by a piston



Piston is shifted by 5 cm in the cylinder

For 1, $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{pV}{300} = \frac{p_2 (V + Ax)}{T_2}$ ...(i)

For 2, $p_1 V_1 = p_2 V_2 \Rightarrow pV = p_2 (V - Ax)$

$$p_2 = \frac{pV}{(V - Ax)} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{pV}{300} = \frac{pV}{(V - Ax)} \cdot \frac{(V + Ax)}{T_2}$$

$$\Rightarrow \quad T_2 = \frac{V + Ax}{V - Ax} 300 = \frac{A45 + A_S}{A45 - AS} \times 300 = \frac{50}{40} 300$$

$$T_2 = 375\text{K}$$

Putting T_2 in Eq. (i),

$$P_2 = \frac{PV}{(V + Ax)} \times \frac{375}{300} = \frac{1 \times A \times 45}{A(450 + 5)} \times \frac{375}{300}$$

$$= 1.125 \text{ atm}$$

37. *(a)* According to thermodynamics first law,

$\Delta Q = \Delta U + \Delta W$

Given, that

$$\Delta Q = +1500 \text{ J}$$

$$\Delta W = +2500 \text{ J}$$

$\therefore$ $1500 = \Delta V + 2500$

$\Rightarrow$ $\Delta V = 1000$ J

$\because$ $\Delta V = m C_V \Delta T$ ...(i)

$\because$ For monoatomic gas,

$$C_V = \frac{3}{2} R = 1.5 \times 8.314$$

$$C_V = 12.471$$

$\therefore$ From Eqs.(i),

$\therefore$ $-1000 = 5 \times 12.471 \times (T_2 - T_1)$

or $T_2 = 150 - \frac{1000}{5 \times 12.471} = 134^\circ \text{C}$

38. *(a)* $\because$ rms speed of molecule $V_{\text{rms}} \propto \sqrt{T}$

for $T = 100\text{K}$

$$(V_{\text{rms}})_1 \propto \sqrt{100}$$

for $T = 400\text{K}$

$$(V_{\text{rms}})_2 = \sqrt{400}$$

therefore, $\frac{(V_{\text{rms}})_1}{(V_{\text{rms}})_2} = \frac{\sqrt{100}}{\sqrt{400}}$

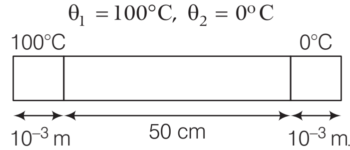
or $\frac{v}{(V_{\text{rms}})_2} = \frac{1}{2}$

or $(V_{\text{rms}})_2 = 2v$

39. *(b)* Given, $K_c = 400 \text{ WM}^{-1}\text{K}^{-1}$,

$$K_w = 0.4 \text{ WM}^{-1}\text{K}^{-1}$$

$$\theta_1 = 100°\text{C}, \ \theta_2 = 0°\text{C}$$



In steady state, flow of heat will be same throughout the whole system

$$\frac{K_w A(100 - \theta)}{10^{-3}} = \frac{K_c A(\theta - \theta')}{(50/100)}$$

or $(100-\theta) = \frac{K_c}{K_w} \times 10^{-3} \times \frac{(\theta - \theta')}{0.5}$

$= \frac{400}{0.4} \times 10^{-3} \times \frac{(\theta - \theta')}{0.5}$

$(100-\theta) = 2(\theta - \theta')$

$\Rightarrow (100-\theta) = 2(\theta - 100 + \theta)$ $(\because 100 - \theta = \theta' \text{given})$

or $(100-\theta) = 4\theta - 200$

$\Rightarrow 5\theta = 300 \Rightarrow \theta = 60^\circ C$

$\therefore \theta' = 100 - \theta = 100 - 60 = 40^\circ C$

Hence, temperature gradient along the bar

$$= \frac{\theta - \theta'}{0.5} = \frac{60-40}{0.5} = 40^\circ \text{cm}$$

40. *(b)* Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \times 100$$

When $\eta = 50\%$

then $\frac{50}{100} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \Rightarrow 0.5 = 1 - \frac{350}{T_{\text{hot}}}$

or $T_{\text{hot}} = 700\text{ K}$

When $\eta = 60\%$

then $\frac{60}{100} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \Rightarrow 0.6 = 1 - \frac{350}{T_{\text{hot}}}$

or $T_{\text{hot}} = 875\text{ K}$

Hence, the temperature of high reservoir is increased by

$$T'_{\text{hot}} - T_{\text{hot}} = 875\text{ K} - 700\text{ K} = 175\text{ K}$$

41. *(a)* Energy given by a system = Energy taken by another system

$$dQ_1 = dQ_2$$

$\Rightarrow = \int_{100}^{T} mc_v dT = -\int_{200}^{T} mc_v dT$

$\Rightarrow \int_{100}^{T} bT^3 dT = -\int_{200}^{T} bT^3 dT \Rightarrow (T^4)_{10}^{T} = -(T^4)_{200}^{T}$

$\Rightarrow T^4 - 100^4 = -(T^4 - 200^4)$

$\Rightarrow 2T^4 = 17 \times 10^8 \Rightarrow T = 171\text{ K}$

42. *(e)* Two point are situated in a pipe and their height from ground is zero $(h = 0)$.

$\because$ According Bernoulli's theorem,

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

At a point pressure of water is p and speed of flow is v

therefore, $p + \frac{1}{2}\rho v^2 = c$ ...(i)

At another point speed of flow is 2v therefore,

$p_1 + \frac{1}{2}\rho (2v)^2 = c$ ...(ii)

To find pressure p_1 at another point, equating the equation (i) and (ii)

$\Rightarrow p + \frac{1}{2}\rho v^2 = p_1 + \frac{1}{2}\rho(4v^2)$

$\Rightarrow p_1 = p - \frac{3}{2}\rho v^2$

43. *(c)* Energy spent = Work done = $T\Delta A$

Here, A is total surface area

$\therefore A = 8\pi r^2$ for soap bubble

Hence, energy spent $= \sigma[8\pi(2r)^2 - 8\pi r^2]$

$= \sigma(32\pi r^2 - 8\pi r^2)$

$= 24\pi\sigma r^2$

44. *(e)* $\because$ Mean momentum of a nucleon in a nucleus is proportional to $A^{1/3}$.

45. *(d)*

(i) When α- particle is emitted from a radioactive nucleus then atomic number decreases by 2 and atomic mass decreases by 4.

(ii) When β- particle emitted from a radioactive nucleus then atomic number is increased by 1 and atomic mass unaffected.

Here, atomic number $(Z) = 92$

After emitting 8 α- particles and 6 β- particles,

$$Z = 92 - 2 \times 8 + 6 = 82$$

and atomic mass, $A = 238$

$\therefore A = 238 - 4 \times 8 = 206$

46. *(d)* $\because$ Angular velocity of mirror = 0.4 rev/s

$= 0.4 \times 2\pi = 0.8\pi$ rad/s

$\therefore$ Angular velocity of reflected ray

$= 2 \times 0.8\pi = 1.6\pi$ rad/s

Hence, velocity of light spot over the screen

$v = rw = 15 \times 1.6\pi = 75.4$ m/s

47. *(b)* Angular spread of the central maxima in the diffraction pattern $= \frac{2\lambda}{e}$ (e = width of the slit)

Here, $e = 5 \times 10^{-3}$ m, $\lambda = 4000\text{Å}$

Hence, $2\theta = 2\left(\frac{4000 \times 10^{-10}}{5 \times 10^{-3}}\right)$

$= 1600 \times 10^{-7}$ rad

or $2\theta = 1.6 \times 10^{-4}$ rad

48. *(c)* Given, resistivity $(\rho) = 2.8 \times 10^{-8}\ \Omega$-m,

radius $(r) = 2 \times 10^{-3}$ m,

current $I = 5$A

Voltage difference $(V) = 1v$

$\because$ Resistivity $(\rho) = \dfrac{RA}{l}$

$$\therefore \quad 2.8 \times 10^{-8} = \frac{R\pi(2 \times 10^{-3})^2}{l}$$

$$\Rightarrow \quad l = \frac{R(\pi \times 4 \times 10^{-6})}{2.8 \times 10^{-8}}$$

$$\Rightarrow \quad l = \frac{(V/I)4\pi \times 10^{-6}}{2.8 \times 10^{-8}}$$

(from Ohm's law, $V = IR$)

$$l = \frac{1}{5} \times \frac{4\pi \times 10^{-6}}{2.8 \times 10^{-8}} = 0.897 \times 10^2 = 89.7 \text{ m}$$

$$\approx 90 \text{ m}$$

49. *(c)* When capacitors are connected in parallel

then $\quad C_{eq} = C_1 + C_2 = 10\ \mu\text{F} \quad \ldots\text{(i)}$

When capacitors are connected in series

then $\quad C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = 0.5\ \mu\text{F}$

or $\quad C_1 C_2 = 0.5\ (C_1 + C_2)$

or $\quad C_1 C_2 = 5\ (\mu\text{F})^2 \quad \ldots\text{(ii)}$

$(\because C_1 + C_2 = 10\ \mu\text{F})$

$\because \quad (C_1 + C_2)^2 = C_1^2 + C_2^2 - 2\,C_1 C_2 \quad \ldots\text{(iii)}$

$(C_1 - C_2)^2 = C_1^2 + C_2^2 - 2C_1 C_2 \quad \ldots\text{(iv)}$

Put the value from Eqs. (i) and (ii) in the eq. (iii)

Hence, $\quad C_1^2 + C_2^2 = (10\ \mu)^2 - 2 \times 5\ \mu^2 = 90\ \mu^2$

Put the above value in Eq (iv)

$$(C_1 - C_2)^2 = 90\ \mu^2 - 2 \times 5\ \mu^2 = 80\ \mu^2$$

$\therefore \quad C_1 - C_2 = \sqrt{5}\ \mu \quad \ldots\text{(v)}$

Now from Eqs. (i) and (v),

$$C_1 = \frac{10 + 4\sqrt{5}}{2} = (5 + 2\sqrt{5})\ \mu\text{F}$$

$$C_2 = \frac{10 - 4\sqrt{5}}{2} = (5 - 2\sqrt{5})\ \mu\text{F}$$

50. *(a)* Magnetic field due to a straight current carrying conductor of finite length

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}(\sin\theta_1 + \sin\theta_2) \quad \ldots\text{(i)}$$

(i) Magnetic field due to conductor DA.

Here, $\quad d = \dfrac{4}{2} = 2\text{m}$

$$\theta_1 = \theta_2 = \tan^{-1}\left(\frac{2\sqrt{2}}{2}\right) = 54.73^\circ$$

$\therefore \quad \sin\theta_1 = \sin\theta_2 = \sin 54.73^\circ = 0.816$

and $\quad I = 5A \quad$ (given)

From Eq. (i),

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{5}{2}(0.816 + 0.816)$$

$$B_1 = 4.08 \times 10^{-7}\ \text{T}$$

Similarly, magnetic field due to conductor BC

$B_2 = 4.08 \times 10^{-7}$ T.

(ii) Magnetic field due to conductor AB.

Here, $\quad d = \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$ m

$$\theta_1 = \theta_2 = \tan^{-1}\left(\frac{2}{2\sqrt{2}}\right) = 35.26^\circ$$

$\therefore \sin\theta_1 = \sin\theta_2 = \sin 35.26^\circ = 0.577$

and $\quad I = 5A \quad$ (given)

From Eq. (i),

$$B_3 = \frac{\mu_0}{4\pi} \times \frac{5}{2\sqrt{2}}(0.57 + 0.57)$$

$$B_3 = 2.04 \times 10^{-7}\ \text{T}$$

Similarly magnetic field due to conductor CD

$$B_4 = 2.04 \times 10^{-7}\text{T}$$

Hence, magnitude of induction field vector B at the intersection of the diagonals

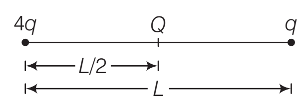
$$B = B_1 + B_2 + B_3 + B_4$$

$$B = (4.08 + 4.08 + 2.04 + 2.04) \times 10^{-7}$$

$$B = 12.24 \times 10^{-7}\ \text{T}$$

or $\quad B = 1.2 \times 10^{-6}$ T

51. *(b)* .



$\because$ Net force on charge, $q = 0$

$\Rightarrow \quad f_1 + f_2 = 0 \quad \ldots\text{(i)}$

Here, f_1 is the force on charge q due to charge $4q$

f_2 is the force on charge q due to charge Q

$\therefore$ From eq. (i),

$$\Rightarrow \quad \frac{K(4q)q}{L^2} + \frac{KQq}{(L/2)^2} = 0$$

$$\Rightarrow \quad 4q^2 + 4Qq = 0$$

$$\Rightarrow \quad Q = -q$$

52. *(c)* $\because$ Lorentz force, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad \ldots\text{(i)}$

Here $\quad \mathbf{v} = a\,\hat{\mathbf{i}},\ \mathbf{B} = b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}},\ q = Q$

From Eq. (i),

$$\mathbf{F} = Q(a\hat{\mathbf{i}} \times b\hat{\mathbf{j}} + c\hat{\mathbf{k}})$$

$$\mathbf{F} = Q(ab\hat{\mathbf{k}} - ac\hat{\mathbf{j}})$$

(According cross production),

$$(\because \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}, \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}})$$

$\therefore$ Magnitude of the force,

$$|\mathbf{F}| = Q\sqrt{(ab)^2 + (ac)^2}$$

or $$\mathbf{F} = Qa\sqrt{(b^2 + c^2)}$$

53. *(d)* The force due to point charge $+Q$ on charge $-q$ is $$F = \frac{kQ(-q)}{R^2}$$

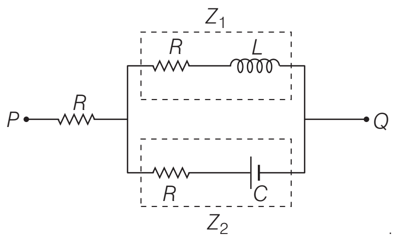
Therefore work required to increase the radius of revolution of $-q$ from R_1 to R_2 is

$$W = -\int_{R_1}^{R_2} F.dR$$

$$= -\int_{R_1}^{R_2} -k\frac{Qq}{R^2}\,dR = kQq\left[\frac{1}{R}\right]_{R_1}^{R_2}$$

$$= -kQq\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

54. *(c)* $\because$ Given that, $C = \frac{1}{\omega R\sqrt{3}}, L = \frac{R\sqrt{3}}{\omega}$



In the above figure,

$$Z_1 = R + j\omega L = R + j\omega\left(\frac{R\sqrt{3}}{\omega}\right) = R + jR\sqrt{3}$$

$$Z_2 = R - j\frac{1}{\omega c} = R - j\frac{1}{\omega\left(\frac{1}{\omega R\sqrt{3}}\right)} = R - jR\sqrt{3}$$

$\because$ Impedance Z_1 and Z_2 are in parallel,

So, $$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(R + jR\sqrt{3})(R - jR\sqrt{3})}{R + jR\sqrt{3} + R - jR\sqrt{3}}$$

$$= \frac{R^2 + R^2(\sqrt{3})^2}{2R}$$

$$(\because (a-b)(a+b) = a^2 - b^2)$$

$$Z_{eq} = \frac{4R^2}{2R} = 2R$$

So, total impedance between P and Q is

$$Z_{PQ} = R + Z_{eq}$$

$$Z_{PQ} = R + 2R = 3R$$

55. *(d)* Given,

$$\frac{E_A}{E_B} = \frac{1}{2}, \frac{U_A}{U_B} = \frac{1}{2}$$

So $E_A = x, E_B = 2x$

and $U_A = y, U_B = 2y$

$\because$ $E_A = U_A + K_A$

and $E_B = U_B + K_B$

here K_A and K_B are kinetic energy of particles A and B

So $$K_A = E_A - U_A = (x - y) \quad \ldots(i)$$

$$K_B = E_B - U_B = 2(x - y) \quad \ldots(ii)$$

$\because$ de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mk}}$$

So $$\lambda_A = \frac{h}{\sqrt{2mK_A}}, \lambda_B = \frac{h}{\sqrt{2mK_B}}$$

$\therefore$ $$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{K_B}{K_A}} \quad \ldots(iii)$$

From Eq. (i), (ii) and (iii),

$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{2(x-y)}{(x-y)}} = \frac{\sqrt{2}}{1}$$

56. *(a)* $\because$ Electrical conductivity, $G \propto \lambda$

Here, λ is mean free path.

57. *(b)* 2 MeV neutron looses half of its kinetic energy in each collision.

So, making geometric progression of each collision 3 MeV, 1 MeV, 5 MeV……0.039 eV.

In this geometric progression,

$$a = 2\text{MeV}$$

$$\gamma = \frac{1}{2}$$

nth term,

$$a_n = ar^{n-1}$$

$\therefore$ $$0.039 = 2\times10^6(1/2)^{n-1}$$

$\Rightarrow$ $$1.95\times10^{-8} = (1/2)^{n-1}$$

Taking log both sides,

$\Rightarrow$ $$(n-1)\ln\left(\frac{1}{2}\right) = \ln(1.95\times10^{-8})$$

Number of collision $(n) = 26.6 \approx 26$ collision

58. *(d)* In Zener diode,

$$\Rightarrow \quad \frac{V_{\max} - V_Z}{R} \geq I_{\text{knee}} + I_2$$

$$\Rightarrow \quad \frac{10-6}{50} \geq 5\,m + I_R$$

$$\Rightarrow \quad I_R \leq 80\text{ m} - 5\text{m}$$

$\because I_{R_{\max}} = 75\,\text{mA}$ and $I_{R_{\min}} = 80\ \text{mA}$

59. *(a)* For electrons to move undeviated along its original path,

$\Rightarrow$ Electric force = Magnetic force

or $\quad qE = qvB$

or $\quad B = \dfrac{E}{v} = \dfrac{8\times10^7}{2\times10^6}$

$B = 40\,\text{T}$

Direction of magnetic field will be – ve z-direction because of negative charge.

60. *(b)* For constructive interference,

$$2\,t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

$$\Rightarrow \quad t = \frac{\left(m+\frac{1}{2}\right)\lambda}{2n} = \frac{\left(0+\frac{1}{2}\right)\times 620\times10^{-9}}{2\times1.3} = 120\ \text{nm}$$

61. *(c)* $\because$ Given, boolean expression is

$$= PQ + PQR + \overline{P}Q + P\overline{Q}R$$
$$= PQ(1+R) + \overline{P}Q + P\overline{Q}R$$
$$= PQ + \overline{P}Q + P\overline{Q}R \qquad (\because 1 + R = 1)$$
$$= Q(P + \overline{P}) + P\overline{Q}R$$
$$= Q + PR\overline{Q} \qquad (\because P + \overline{P} = 1)$$
$$= (Q + PR)(Q + \overline{Q})$$
$$= Q + PR \qquad (\because Q + \overline{Q} = 1)$$

62. *(e)* $\because \quad \mu = \dfrac{\sin\left[\dfrac{(A+\delta_m)}{2}\right]}{\sin\left(\dfrac{A}{2}\right)}$

Here, μ is refractive index δ_m is minimum deviation angle, A is angle of prism.

$$\therefore \quad \sqrt{2} = \frac{\sin\left[\dfrac{A+A}{2}\right]}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \quad \sqrt{2}\sin\frac{A}{2} = \sin A$$

Hence, $A = 90^\circ$

63. *(a)* Given, $\sigma(x) = \sigma_0 \dfrac{l}{\sqrt{x}}$

$\because$ Resistance of the system along the cylindrical axis,

$$R = \int_O^L \frac{\rho(x)}{A}dx$$

$$= \int_O^L \frac{\left(\dfrac{1}{\sigma_0 \dfrac{L}{\sqrt{x}}}\right)}{A}dx \qquad \left[\because \rho(x) = \frac{1}{\sigma(x)}\right]$$

$$= \int_O^L \frac{\sqrt{x}}{\sigma_0 AL}dx = \frac{1}{\sigma_0 AL}\left(\frac{x^{3/2}}{3/2}\right)_O^L = \frac{2}{3}\frac{1}{\sigma_0 AL}(L^{3/2} - O)$$

$$= \frac{2}{3}\cdot\frac{1}{\sigma_0 AL}\times L^{3/2}, \quad R = \frac{2}{3}\cdot\frac{\sqrt{L}}{A\sigma_0}$$

64. *(d)* According Stefan's law, emission rate of a ideal blackbody

$$E \propto T^4$$

For $\quad T = 0^\circ\text{C} = 273\text{K}$

$$E_1 \propto (273)^4 \qquad \ldots(\text{i})$$

For $T = 273^\circ\text{C} = 273 + 273 = 546$ K

$$E_2 \propto (546)^4 \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii),

$$\Rightarrow \quad \frac{E_1}{E_2} = \left(\frac{273}{546}\right)^4 \Rightarrow \frac{R}{E_2} = \left(\frac{1}{2}\right)^4 \qquad (\because E_1 = R)$$

Hence, $E_2 = 16\,R$

65. *(d)* $\because$ Relation between R, T and A is

$$A + T + R = 1 \qquad \ldots(\text{i})$$

$\because$ For ideal blackbody,

$$A = 1$$

$\therefore$ According Eq. (i),

$$R = 0,\ T = 0 \text{ and } A = 1$$

66. *(d)* From the given circuit, the output Y is

$$Y = (\overline{P} + q)\,(P + q)$$
$$Y = \overline{P}.P + \overline{P}Q + Q.P + Q.Q$$
$$Y = Q + PQ + \overline{P}Q \qquad (\because \overline{P}.P = 0)$$
$$Y = Q(1 + P + \overline{P}) \qquad (\because 1 + A = 1)$$
$$Y = \overline{Q}$$

67. *(e)* Power of transmitted radiation, $P_T = 60$ W.

$\therefore$ Intensity of wave, $I = \dfrac{P_T}{A}$

$\because \quad A = 4\pi R^2$

$4\pi(12\text{Km})^2 \qquad (\because R = 12\text{km given})$

$\Rightarrow \quad A = 4\pi\times144\times10^6$

$$\therefore \quad I = \frac{60}{4\pi\times144\times10^6}$$

$$= 3.31\times10^{-8}\ \text{W/M}^2$$

68. *(a)* $\because$ Speed of sound, $v_{sound} = \sqrt{\frac{xRT}{m}}$

$\therefore \quad = \frac{\gamma RT}{v_{sound}^2}$

For diatomic gas, $\gamma = 1.40$

$\therefore \quad M = \frac{1.4 \times 8.314 \times 273}{(1260)^2} = 2 \times 10^{-3}$ kg = 2 gm

69. *(d)* $\because$ Kinetic energy of satellite,

$$KE = \frac{GMm}{2R}$$

$$\therefore \quad KE \propto \frac{1}{R}$$

$\therefore$ option (d) is correct.

70. *(e)* $\because$ Magnetic field due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$

B can be doubled by changing I to $2I$ and Keeping N and R unchanged.

$\therefore$ Options (e) is correct.

71. *(e)* Given that, $V = V_0 \sin \omega t$

$$I = I_0 \sin(\omega t + II_2)$$

$\because$ Power conumed per cycle

$$P_{av} = V_{rms} \cdot I_{rms} \cos$$

Here $\quad \phi = \frac{\pi}{2}$

$$\therefore \quad P_{av} = V_{rms} . I_{rms} \cos \frac{\pi}{2} = 0 \text{ W}$$

72. *(c)* $\because$ Intensity, $I = \frac{P}{A} = \frac{\text{watt}}{\text{m}^2}$

Speed of light, $c = \frac{m}{s}$

$$\therefore \quad \frac{I}{c} = \frac{\text{watt}}{\text{m}^2 \times \frac{\text{m}}{\text{sec}}} = \frac{\left(\frac{\text{newton} \times \text{m}}{\text{s}}\right)}{\left(\frac{\text{m}^2 \times \text{m}}{\text{s}}\right)} = \frac{\text{newton}}{\text{m}^2}$$

Hence, $\frac{I}{C}$ = pressure.

Chemistry

73. *(c)* Due to small size $(n = 1)$ and fully filled inert gas configuration. He show highest IE.

74. *(d)* The option (d) is false as d-electrons do not filled monotonically with the increase in atomic number.

75. *(c)* Electronegativity increases on moving left to right in a period and decreases for the period below it ($\because n$ = increases)

$\because$ (N and C) and (Si and P) respectively belongs to $(n = 2)$ and $(n = 3)$

$\therefore$ Electronegativity of N > electronegativity of C and electronegativity of P > electronegativity of Si

Hence, correct order is:

$$\text{Si} < \text{P} < \text{C} < \text{N}$$

76. *(d)* The element Gd$(Z = 64)$ show electronic configuration = $[\text{Xe}]_{54} \cdot 4f^7, 5d^1, 6s^2$

Thus, it has 8-unpaired electrons ($4f^7$ and $6d^1$) and its sum of spin is 4.

77. *(c)* On passing the $SO_2(g)$ in the aqueous solution of Na_2CO_3 we get $NaHSO_3$ as product.

$$Na_2CO_3 + H_2O + 2SO_2(g) \longrightarrow 2NaHSO_3 + CO_2$$

78. *(d)* Main components of portland-cement are

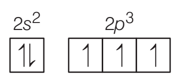
(i) $CaSiO_3$ (ii) $CaSiO_4$

(iii) $CaAl_2O_6$

and some other substance but it does not contain $Ca(PO_4)_2$.

79. *(b)* $Al_2(SO_4)_3$ is not used in plastic industry.

80. *(e)* The outermost orbital of N-atom contain 3 unpaired electrons and has $n = 2$

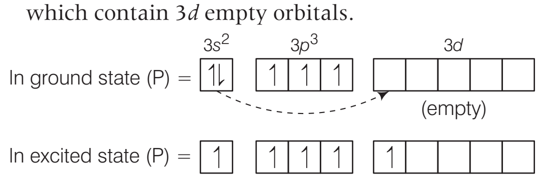


Thus, it is not able to expand its octet and can only form 3 covalent bonds.

It can also form one coordinate bond with the help of one lone-pair of electrons over N-atom ($\therefore$ total = 4)

On the other hand, the outer most orbital for P is $n = 3$

which contain $3d$ empty orbitals.



(has 5-unpaired electrons)

Thus, none of the given are correct.

$\therefore$ Correct choice is (e).

81. *(b)* Only the following statements are correct for N_2H_4.

1. It is an exothermic compound.
2. It burns with evolution of heat.

$\therefore$ (b) is the correct choice.

82. *(a)* $\because$ Among the given species, all are di-atomic species. Only $[O_2]^{2-}$ contain one σ bond between two bonded atoms.

$\therefore$ (a) is the correct option.

83. *(a)* Among the given statements:

1. Cl_2O and ClO_2 give nascen oxygen, thus behave as bleaching agent.
2. Salts of OCl^- are used as detergents.
3. Oxidation state of Cl in OCl^- is (+ 1), thus it shows disproportionation reaction in acidic medium:

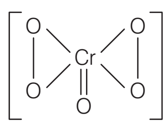
$$\underset{+1}{5ClO^-}(5HClO) \longrightarrow \underset{[0]}{2Cl_2} + \underset{[+5]}{ClO_3^-}(HClO_3) + 2H_2O$$

4. BrO_3^- does not oxidise in acidic medium.

Thus (a) is the correct answer.

84. *(c)* On adding H_2O_2 in acidic solution of $K_2Cr_2O_7$, H_2O_2 oxidise it to CrO_5 (chromic penta oxide/ chromic per oxide) and gives the blue-violet solution.

$$K_2Cr_2O_7 + H_2SO_4 + 4H_2O_2 \longrightarrow K_2SO_4 + 2CrO_5 + 5H_2O$$



(CrO_5)

CrO_5 on decomposition gives oxygen

$$4CrO_5 + 6H_2SO_4 \longrightarrow Cr_2(SO_4)_3 + 6H_2O + 7O_2$$

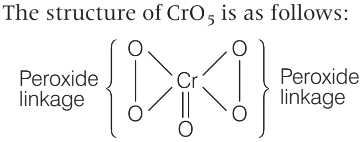
85. *(a)* Only $(NH_4)_2Cr_2O_7$ and NH_4NO_2 give N_2 on heating.

(i) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + 4H_2O + N_2$

(ii) $NH_4NO_2 \xrightarrow[\Delta]{\text{Thermal decomposition}} 2\ H_2O + N_2$

$\therefore$ (a) is the correct option.

86. *(b)* The structure of CrO_5 is as follows:



$\therefore$ It has two per-oxide linkage.

87. *(e)* The elements belong to carbon family are:

Carbon—(C)
Silicon—(Si)
Germanium—(Ge)
Tin—(Sn)
Lead—(Pb)
Flerovium—(Fl)

Except carbon all other elements contain empty d-orbitals in their outermost shell, thus can form more than four-bonds.

$\therefore$ (e) is the correct answer.

88. *(e)* $\because$ Effective nuclear charge = 2.60

and valency shell contain 3 electrons.

Thus, minimum number of main shells for the given element are two i.e $(n=2)$ and its configuration will be $1s^2, 2s^2 2p^1$.

Thus, the given element has 5 electrons in all.

Also, for a neutral atom.

$\because$ No. of electrons = Atomic number

Thus, atomic number of the element is 5.

89. *(a)* Alumina column having suitable solvent elutes the species based on their nature of polarity. Less polar species adsorb first and more polar there after. Thus the correct order is:

Anthracene $\rightarrow$ chlorobenzene $\rightarrow$ p-cresol

90. *(c)* The formula C_6H_4 of alkane gives the following structural isomers—

(a) $CH_3—CH_2—CH_2—CH_2—CH_2—CH_3$
(n-hexane)

(b) $CH_3—CH_2—\underset{\underset{CH_3}{|}}{CH}—CH_2—CH_3$
(3-methyl pentane)

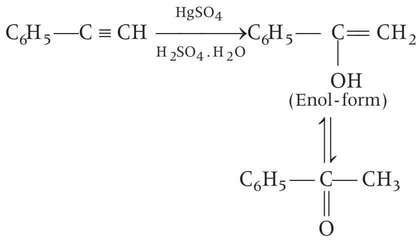
(c) $CH_3—CH_2—CH_2—\underset{\underset{CH_3}{|}}{CH}—CH_3$
(2-methyl pentane)

(d) $CH_3—\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}}—CH_2—CH_3$
(2, 2 dimethyl butane)

(e) $CH_3—\underset{\underset{CH_3}{|}}{CH}—\underset{\underset{CH_3}{|}}{CH}—CH_3$
(2, 3, di-methyl butane)

$\therefore$ Total = 5-isomers

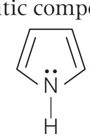
91. *(a)* On reaction with $HgSO_4/H_2SO_4 \cdot H_2O$, phenyl acetylene gives acetophenone as a product:



Acetophenone
(Keto form)

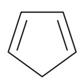
92. *(a)* Structure (a) has two π-bonds and one lone pair of electrons over N-atom and has planer structure.

Thus, is an aromatic compound.

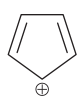


(b) The structure (b) also contain two π-bonds and one lone pair of electrons. Thus, follow Huckel's rule. The structure is cyclic and planer.

∴ (b) is also an aromatic compound.



(c) The structure (c) has only 2π-bonds and do not follow Huckel's rule. So, it not an aromatic compound.



(d) Structure (d) also contain only two π-bonds, thus do not follow Huckel's rule and is not an aromatic compound.

∴ (a) is the correct answer.

93. *(c)* Among the aromatic electrophilic substitution reactions. Sulphonation is an example of reversible reaction.

94. *(c)* *R*—*S*— configuration is related to the enantiomers, which are optically active. (∴ true)

(b) The mixing of two optically-active compounds (*d* and *l*-type) in equimolar quantity is called racemisation (∴ true)

(c) A molecule containing plane of symmetry does not show optical-activity.

Hence, the given statement is false.

(d) Optical isomers that are not enantiomers are called diastereoisomers (is true).

(e) All chiral objects are asymmetric (true).

∴ (c) is the correct answer.

95. *(c)* Neopentyl bromide give a carbocation as intermediate which undergo for rearrangement and show E_1 mechanism (even has no β-H atom). Thus (c) is the correct option.

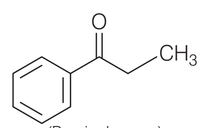
96. *(d)* Chlorobenzene does not lead to nitrile by substitution with NaCN/DMSO due to resonance and double bond character between Cl and carbon [C-atom] of the benzene ring.

97. *(c)* The oxidation of *n*-propyl alcohol (among the 1° alcohols) is very successful because of least steric hindrance in the given molecule.

98. *(e)* The species containing

—$C(CH_3) \cdot (H)(OH)$ or CH_3 >C = O will show haloform reaction.

Thus, propiophenone does not show haloform reaction.



(Propiophenone)

99. *(d)* Due to resonance structure [CH_2 = CH (—Cl)] (vinyl chloride) will not react with phenol to give ethers.

100. *(a)* As pK_a value of

$$pK_a(CH_3COOH) = 4.76$$
$$pK_a(CH_2Cl \cdot COOH) = 2.75$$
$$pK_a(CCl_3COOH) = 0.65$$
$$pK_a\left(CH_3\underset{\underset{O}{\|}}{C}—O—O—H\right) = 8.2$$

and more be the value of pK_a, weaker be the acid. Hence, peroxyacetic acid is the weakest acid.

101. *(e)* On attacking at

N—N— dimethyl aniline by $\overset{+}{N}O_2$, electrophilic nitration takes place and the process is called

Nitrosation. Here $\overset{+}{N}O_2$ act as an attacking electrophilic agent. It is produced as follows:

$$HNO_3 + 2H_2SO_4 \longrightarrow \overset{+}{N}O_2 + 2HSO_4^- + H_2O$$

102. *(d)* In RNA, nitrogen-base uracil is present in place of thymine (which is present in DNA).

∴ (d) is the correct answer.

103. *(a)* Bio-waste give the most efficient and clean fuel thus known as Green-fuel.

104. *(a)* Barbiturates are derivatives of barbituric acids. These are potent hypnotics.

105. *(a)* Atomic mass of Fe = 55.84

$$\because \text{Equivalent mass} = \frac{\text{Atomic mass}}{\text{Change in oxidation state}}$$

For the charge, $Fe^{2+} \longrightarrow Fe^{3+}$ i.e (3 − 2 = 1)

$$\text{the equivalent mass} = \frac{55.84}{1} = 55.84$$

106. *(a)* Molecular mass of C_2H_5OH = 46.00

$$\because \begin{bmatrix} \text{Atomic mass of C} = 12.00 \\ \text{Atomic mass of O} = 16.00 \\ \text{Atomic mass of H} = 1.00 \end{bmatrix}$$

Also,

∵ 46.00 of C_2H_5OH contain, C = 24 g.

$$\therefore 100 \text{ g of } C_2H_5OH \text{ contain, } C = \frac{24 \times 100}{46}$$

(52.17% = 52%)

107. *(b)* The related equation for the formation of ethanol is.

$$2C(\text{graphite}) + 3H_2(g) + \frac{1}{2}O_2(g) \longrightarrow C_2H_5OH(l);$$

$$\Delta H_f = -277.7 \text{ kJ/mol}$$

When 2 moles of graphite is involved the standard enthalpy of reaction is − 277.7 kJ/mol

∴ When 4 moles of graphite is involved, standard enthalpy of reaction is $2 \times (-277.7 \text{ kJ/mol})$

= (–) 555.4 kJ/mol

108. *(e)* The energy required to separate one mole CH_4 molecule into its gaseous atoms is known as heat of atomisation.

In this process, heat is absorbed i.e. (endothermic) and the said reaction is non-spontaneous.

∴ (*e*) is the correct answer.

109. *(b)* Given,

$$X_2 + Y_2 \longrightarrow ?P$$

Let the coefficient of *P* be *x*.

Now, we know that

$$K = \frac{[P]^x}{[X_2][Y_2]}$$

According to the data given in question,

$$K_1 = \frac{(2.52 \times 10^{-2})^x}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})}$$

$$\text{Also, } K_2 = \frac{(3.08 \times 10^{-2})^x}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})}$$

Now, substitute the values of *x* given in option one by one.

(a) $$K_1 = \frac{(2.52 \times 10^{-2})}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})} = 1842.10$$

$$K_2 = \frac{(3.08 \times 10^{-2})}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})} = 1521.73$$

∵ $K_1 \neq K_2$, thus option (a) is incorrect.

(b) $$K_1 = \frac{(2.52 \times 10^{-2})^2}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})} = 46.42$$

$$K_2 = \frac{(3.08 \times 10^{-2})^2}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})} = 46.36$$

∵ $K_1 \neq K_2$

∴ Option (b) is correct.

110. *(a)* ∵ $\Delta G°$ (Gibbs free energy) is related to *K* (equilibrium constant) as follows:

$$\Delta G° = -2.303 \, RT \log_{10} K$$

where, *R* = Gas constant

T = Temperature in Kelvin

By knowing the value of *K* we can find out rate of a reaction.

(*a*) is the correct answer.

111. *(b)* Total charge produced by cell

$$= 1 \text{ A} \times (595.1 \times 3600) \text{ s}$$

$$= 2142360 \text{ C}$$

$$\because \quad 96500 \text{ C} = 1 \text{ mol}$$

$$\therefore \quad 2142360 \text{ C} = \frac{2142360}{96500}$$

$$= 22.20 \text{ mol of } e^-$$

Now, in H_2-O_2 fuel cell following reaction occurs,

At anode $2H_2(g) + 4O^-H(aq) \longrightarrow 4H_2O(l) + 4e^-$

At cathode $O_2(g) + 2H_2O(l) + 4e^- \longrightarrow 4OH^-(aq)$

Overall reaction $2H_2(g) + O_2(g) \longrightarrow 2H_2O(l)$

Thus, from above reaction it is clear that

$$2 \text{ mol of } e^- \equiv 1 \text{ mol of } H_2O$$

$\because$ 22 mol of $e^- \equiv$ 11 mol of H_2O

$= (11 \times 18)$ g of H_2O

$\left[\because \text{No. of moles} = \frac{\text{Weight}}{\text{Molecular weight}}\right]$

$= 198$ g or mL of H_2O

Now, number of mol of NaOH $= \frac{5}{40} = 0.125$ mol

We know that,

$$\text{Molarity} = \frac{\text{No. of moles of solute}}{\text{Volume of solution (in L)}}$$

$$= \frac{0.125}{198} \times 1000 = 0.63\,\text{M}$$

112. *(b)* $\because \Delta T = K_b \frac{n}{w_A}$

Given, $K_b = 0.52$ K kg mol^{-1}

$n = 1, \quad w_A = 1$ kg

$\therefore \quad \Delta T = \frac{0.52 \times 1}{1} = 0.52$ K

$\because$ Boiling point of pure water at 1.013 bar (i.e. 1 atm) is 373.15 K

$\therefore$ Boiling point of the solution $= 373.15 + 0.52 = 373.67$ K

113. *(a)* The electrochemical reaction between Ag(s) and $Cl_2(g)$ is as follows:

$$AgCl \rightleftharpoons Ag^+ + Cl^-$$

Given, $K_{sp} = 1.8 \times 10^{-10}$, $[Cl^-] = 0.1$ M

$\therefore \quad K_{sp} = [Ag^+][Cl^-]$

$1.8 \times 10^{-10} = [Ag^+] \times 0.1$

$\therefore [Ag^+] = \frac{1.8 \times 10^{-10}}{0.1} = 1.8 \times 10^{-9}$ M

$\therefore$ 1 L of solution contains 1.8×10^{-9} moles of Ag^+.

Quantity of electricity required

$= 1.8 \times 10^{-9} \times 96500$

$= 1.73 \times 10^{-4}$ C

$\therefore$ Time required $(t) = \frac{1.73 \times 10^{-4}}{1 \times 10^{-6}} = 173$ s

114. *(a)* Now, the cell reaction is

$$Li\,(s) + \frac{1}{2}F_2(g) \longrightarrow Li^+ + F^-$$

We know that,

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln\frac{[\text{Product}]}{[\text{Reactant}]}$$

$$= E^\circ_{cell} - \frac{2.303\,RT}{nF}\log[Li^+][F^-]$$

$$= 5.92 - \frac{2.303 \times 8.314 \times 298}{1 \times 96500}\log(2 \times 2)$$

$$= 5.92 - \frac{0.059}{1} \times 2\log 2$$

$$= 5.92 - 0.035 = 5.88\text{ V}$$

115. *(a)* Given, $\Lambda_m = 240$ S cm^2 mol^{-1},

$\Lambda_m^\infty = 240$ Scm^2 mol^{-1}

$K_b = 0.52$ K kg mol^{-1}

Degree of ionisation, $\alpha = \frac{\Lambda_m}{\Lambda_m^\infty}$

$$= \frac{240\text{ Scm}^2\text{mol}^{-1}}{420\text{ Scm}^2\text{mol}^{-1}} = 0.57$$

van't Hoff factor

$i = 1 + (n-1)\alpha$ [for ionisation]

$= 1 + (2-1)\,0.57$ [for HCl, $n = 2$]

$= 1.57$

Elevation in boiling,

$\Delta T_b = iK_b \times \text{molality } (m)$

$= 1.57 \times 0.52$ K kg mol^{-1} $\times 3$ mol kg^{-1}

$= 2.45$ K

Since, water boils at 373.15 K at 1 bar pressure, therefore the boiling point of solution will be

$T_b = T_b^\circ + \Delta T_b = 373.15 + 2.45$

$= 375.6$ K

116. *(a)* From the data (1) and (2)

$$\frac{[r_{D_2}]_2}{[r_{D_2}]_1} = \frac{K[D_2]_2}{K[D_2]_1}$$

i.e $\quad \frac{3 \times 10^{-3}}{1 \times 10^{-3}} = \frac{[0.15]}{[0.05]}$

$(3) = [3]^n$

$\because$ Rate $(r) \propto [\text{concentration}]^n$, where n = order

$\therefore \quad r(D_2) = [3]^1$

$\therefore$ Order of $[D_2] = 1$

Similarly, from data (1) and (3)

$$\frac{(r_A)_3}{(r_A)_1} = \frac{k[A]_3}{k[A]_1}$$

$\Rightarrow \quad \frac{9 \times 10^{-3}}{1 \times 10^{-3}} = \frac{0.15}{0.05} = 3$

$\because$ rate $(r) = [\text{conc.}]^n$

$\therefore \quad a = [3]^2$

or order for $[A] = 2$

Hence, rate expression $= k[D_2]^1 \cdot [A]^2$

$\therefore$ (a) is the correct answer.

117. *(a)* $\because$ Rate (r) $= k$ [concentration]^n

$\therefore \quad k = \dfrac{\text{rate}}{[\text{concentration}]^n}$

or, $\quad k = \dfrac{[\text{mol L}^{-1}]\cdot \text{s}^{-1}}{[\text{mol L}^{-1}]^{\frac{1}{2}}\cdot[\text{mol L}^{-1}]^{\frac{3}{2}}}$

$k = \dfrac{[\text{mol L}^{-1}]\cdot \text{s}^{-1}}{[\text{mol L}^{-1}]^2}$

$k = \text{mol}^{-1}\text{L s}^{-1}$

$\therefore$ (a) is the correct answer.

118. *(b)* The reaction,

$$2\text{HI} \xrightarrow{\Delta,\ \text{catalyst}} H_2(g) + I_2(g)$$

is of zero-order in which HI is present at high partial pressure.

119. *(a)* In adsorption the adsorbed particles show strong force of attraction with the surface on which they adsorb and therefore their randomness also decreases.

$\therefore \quad \Delta H < 0$ and $\Delta S < 0$

Hence, (a) is the correct answer.

120. *(a)* In formation of NH_3 by Haber's process.

$$N_2(g) + 3H_2(g) \xrightarrow{\Delta,\ \text{catalyst}} 2NH_3(g)$$

(i) When Mo is used as catalyst, it increase the rate of formation of NH_3 because it behaves as promoter.

(ii) But, when CO is used as catalyst it decreases the formation of NH_3 because it behaves as poisoning agent.

$\therefore$ (a) is the correct answer.

Mathematics

1. *(b)* $\dfrac{2(\cos 75° + i\sin 75°)}{0.2(\cos 30°\ \ i\sin 30°)} = \dfrac{2\cdot e^{i\,75°}}{0.2\cdot e^{i\,30°}}$

$(\because \cos\theta + i\sin\theta = e^{i\theta})$

$= 10\cdot e^{i75°}\cdot e^{-i30°}$

$= 10\cdot e^{i\,45°}$

$= 10(\cos 45° + i\sin 45°)$

$(e^{i\theta} = \cos\theta + i\sin\theta)$

$= 10\left(\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{10}{\sqrt{2}}(1 + i)$

2. *(d)* $z = \dfrac{1}{i-1}\times\dfrac{i+1}{i+1}$

$\Rightarrow \quad z = \dfrac{i+1}{i^2 - 1^2}$

$z = -\dfrac{1}{2}\times(i+1)$

$\Rightarrow \quad \bar{z} = -\dfrac{1}{2}(1-i)\times\dfrac{(1+i)}{(1+i)}$

$= -\dfrac{1}{2}\dfrac{(1+1)}{(1+i)} = -\dfrac{1}{(1+i)}$

3. *(e)* $\left\{i^{18} + \left(\dfrac{1}{i}\right)^{25}\right\}^3 = \left\{(i^4)^4\cdot i^2 + \left(\dfrac{1}{i^4}\right)^6\cdot\dfrac{1}{i}\right\}^3$

$= \left[1\cdot(-1) + 1\cdot\dfrac{1}{i}\right]^3$

$= \left[\dfrac{1}{i} - 1\right]^3$

$= \dfrac{1}{i^3} - 1 + \dfrac{3}{i}\left(1 - \dfrac{1}{i}\right)$

$= i - 1 - 3i + 3$

$= 2 - 2i$

4. *(a)* $\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} = \dfrac{(1+i)^2 - (1-i)^2}{1^2 - i^2}$

$= \dfrac{1 + i^2 + 2i - 1 - i^2 + 2i}{2} = \dfrac{4i}{2}$

$= 2i = 0 + 2i$

Modulus of $\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} = |0 + 2i|$

$= \sqrt{0^2 + 2^2} = \sqrt{4} = 2$

5. *(b)* $z = e^{\frac{i\,4\pi}{3}}$

$z = \left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

$z = -\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$

$z = \omega^2$

$(z^{192} + z^{194})^3 = [(\omega^2)^{192} + (\omega^2)^{194}]^3$

$= [\omega^{384} + \omega^{388}]^3$

$= [(\omega^3)^{128} + (\omega^3)^{129}\cdot\omega]^3$

$= (1 + \omega)^3$

$= 1 + \omega^3 + 3\omega^2 + 3\omega$
$= 1 + 1 + 3(\omega + \omega^2)$
$= 1 + 1 + 3(-1)$
$= 1 + 1 - 3$
$= -1$

6. *(e)* Given, $(a + ib)^{11} = 1 + 3i$

So, $(a - ib)^{11} = 1 - 3i$ ...(i)

Then, $(b + ia)^{11} = (i)^{11}\left\{\frac{b}{i} + a\right\}^{11}$

$= (i)^{11}\{-bi + a\}^{11}$

$= -i(a - ib)^{11}$

From Eq. (i), we get

$= -i(1 - 3i)$

$= -i + 3i^2$

$= -i - 3$

7. *(e)* Given, $\alpha^2 = 5\alpha - 3$

and $\beta^2 = 5\beta - 3$

$\alpha^2 - 5\alpha + 3 = 0$

$\Rightarrow \alpha = \frac{5 \pm \sqrt{25-12}}{2}$

$= \frac{5 \pm \sqrt{13}}{2}$

Similarly, $\beta = \frac{5 \pm \sqrt{13}}{2}$

$\because \alpha \neq \beta$

$\therefore \alpha = \frac{5 + \sqrt{13}}{2}, \beta = \frac{5 - \sqrt{13}}{2}$

Now, addition of roots

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{5 + \sqrt{13}}{5 - \sqrt{13}} + \frac{5 - \sqrt{13}}{5 + \sqrt{13}} = \frac{19}{3}$

Multiplication of roots $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$\therefore x^2 - \left(\frac{19}{3}\right)x + 1 = 0$

$\Rightarrow 3x^2 - 19x + 3 = 0$

8. *(d)* $y^2 - 4y - x + 3 = 0$

$(y - 2)^2 - 4 - x + 3 = 0$

$(y - 2)^2 - x - 1 = 0$

$(y - 2)^2 = (x + 1)$

Let $Y^2 = X$ ...(i)

Here, $Y = (y - 2), X = (x + 1)$

Vertices $(X = 0, Y = 0) = (2, -1)$

Eq. (i) comparing on $y^2 = 4ax$

$4a = 1$

$\Rightarrow a = \frac{1}{4}$

$\therefore$ Focus $= \left(\frac{1}{4} - 1, 2\right) = \left(-\frac{3}{4}, 2\right)$

9. *(e)* Given, $f : R \to (0, \infty)$ is an increasing function.

And $\lim_{x \to 2018} \frac{f(3x)}{f(x)} = 1$

So, $\lim_{x \to 2018} f(3x) = \lim_{x \to 2018} f(x)$

$\Rightarrow f(x) =$ constant.

Therefore, $\lim_{x \to 2018} \frac{f(2x)}{f(x)} = 1$

10. *(c)* $\lim_{x \to 1} \frac{x^2 f(1) - f(x)}{x - 1}$

By L' Hospital's Rule,

$\lim_{x \to 1} \frac{2x\, f(1) - f'(x)}{1 - 0} = 2f(1) - f'(1)$

11. *(a)* Equation $4x^2 + y^2 - 8x + 4y - 8 = 0$ is an ellipse.

$\Rightarrow 4(x - 1)^2 + (y + 2)^2 - 8 - 8 = 0$

$= 4(x - 1)^2 + (y + 2)^2 = 16$

$= \frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{16} = 1$, where $b > a$

$\therefore$ Eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$

$= \sqrt{1 - \frac{4}{16}} = \sqrt{\frac{12}{16}} = \frac{\sqrt{3}}{2}$

12. *(a)* Given, $(y + 1)^2 = -8(x + 2)$

$Y^2 = -8X$ ...(i)

Here, $Y = y + 1$, $X = (x + 2)$

Vertices $(X = 0, Y = 0) = (-2, -1)$

Comparing Eq. (i) from $y^2 = 4ax$

$4a = -8$

$a = -2$

Focus $= (-2 - 2, -1)$

$= (-4, -1)$

13. *(d)* $x^2 - y^2 + 3x - 2y - 43 = 0$

$= \left(x + \frac{3}{2}\right)^2 - (y + 1)^2 - \frac{5}{4} - 43 = 0$

$$= \left(x + \frac{3}{2}\right)^2 - (y+1)^2 = \frac{177}{4}$$

$$= \frac{\left(x + \frac{3}{2}\right)^2}{\frac{177}{4}} - \frac{(y+1)^2}{\frac{177}{4}} = 1$$

It is hyperbola equation.

14. *(a)* $f(x) = px^2 + qx + r$

$f(-4) = 0$

$\Rightarrow \quad 16p - 4q + r = 0 \quad \ldots(i)$

One root is $x = -4$

and $f(5) = -3f(2)$

$25p + 5q + r = -3(4p + 2q + r)$

$\Rightarrow \quad 37p + 11q + 4r = 0 \quad \ldots(ii)$

Eq. (ii) – Eq. (i), we get

$\Rightarrow \quad -27p + 27q = 0$

$\Rightarrow \quad p = q$

Then, equation is $px^2 + qx + r = 0$

Roots $= -4, \alpha$

Sum of roots $= -4x + \alpha = -\frac{p}{q} = -1$

So, another root $\alpha = 3$.

15. *(b)* Given, $f : \bullet \to \bullet$

$f(x)\, f(y) = f(xy) \quad \ldots(i)$

On taking $x = 1, y = 1$

$f(1)\, f(1) = f(1 \cdot 1) = f(1)^2 = f(1) = f(1) = 1$

Now, $x = 2$, $y = \frac{1}{2}$, then from Eq. (i)

$$f(2)\, f\left(\frac{1}{2}\right) = f\left(2 \cdot \frac{1}{2}\right)$$

$\Rightarrow \quad 4 \cdot f\left(\frac{1}{2}\right) = f(1) \quad [\because f(2) = 4]$

$\Rightarrow \quad f\left(\frac{1}{2}\right) = \frac{1}{4} f(1)$

On putting the value of $f(1)$,

$\Rightarrow \quad f\left(\frac{1}{2}\right) = \frac{1}{4} \cdot 1 = \frac{1}{4}$

16. *(c)* We have, $(1 + x)^{59}$

Sum of last 30 coefficient of the binomial expansion

$= {}^{59}C_{30} + {}^{59}C_{31} + \ldots + {}^{59}C_{59}$

We know that,

${}^{59}C_0 + {}^{59}C_1 + {}^{59}C_2 + \ldots + {}^{59}C_{59} = 2^{59}$

$\Rightarrow ({}^{59}C_0 + {}^{59}C_{59}) + ({}^{59}C_1 + {}^{59}C_{58}) + \ldots.$

$+ ({}^{59}C_{29} + {}^{59}C_{30}) = 2^{59}$

$\Rightarrow 2({}^{59}C_{59} + {}^{59}C_{58} + \ldots + {}^{59}C_{31} + {}^{59}C_{30}) = 2^{59}$

$[\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow \quad {}^{59}C_{30} + {}^{59}C_{31} + \ldots {}^{59}C_{39} = \frac{2^{59}}{2} = 2^{58}$

$\therefore$ Sum of last 30 coefficient of the binomial expansion $(1 + x)^{59}$ is 2^{58}.

17. *(d)* Take,

$(a + b)^4 = {}^4C_0\, a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 ab^3 + {}^4C_4 b^4$

$= {}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_1 ab^3 + {}^4C_0 b^4$

$(\because {}^nC_r = {}^nC_{n-r})$

$= 1 \times a^4 + 4a^3 b + \frac{4 \times 3}{2} a^2 b^2 + 4ab^3 + 1 \times b^4$

$\Rightarrow \quad (a + b)^4 = a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + b^4 \quad \ldots(i)$

Similarly, $(a - b)^4 = a^4 - 4a^3 b + 6a^2 b^2 - 4ab^3 + b^4 \quad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$(a + b)^4 - (a - b)^4 = 8a^3 b + 8ab^3$

$= 8ab(a^2 + b^2)$

Now, putting $a = \sqrt{3}$ and $b = \sqrt{2}$

$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8\sqrt{3}\sqrt{2}\,[(\sqrt{3})^2 + (\sqrt{2})^2]$

$= 8\sqrt{6}(3 + 2) = 8\sqrt{6} \times 5 = 40\sqrt{6}$

18. *(d)* We have, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

$\therefore$ Required probability

$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

19. *(b)* Given, $z_1 = 2 - i$ and $z_2 = 1 + i$

Then, $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = \left|\frac{2 - i + 1 + i + 1}{2 - i - 1 - i + i}\right|$

$= \left|\frac{4}{(1 - i)} \times \frac{(1 + i)}{(1 + i)}\right|$

$= \left|\frac{4(1 + i)}{2}\right|$

$= |2 + 2i|$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{8} = 2\sqrt{2}$

20. *(a)* Given, $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$

Now, $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$

$= \lim_{x \to \infty} \sqrt{\frac{1 - \frac{\sin x}{x}}{1 + \frac{\cos^2 x}{x}}}$

$= \sqrt{\frac{1-0}{1+0}} \quad \left(\because \frac{\sin x}{x} \to 0, \frac{\cos^2 x}{x} \to 0 \text{ as } x \to \infty\right)$

$= 1$

21. *(a)* $\sin\frac{31}{3}\pi = \sin\left(10\pi + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

22. *(c)* $1 + 3 + 5 + \dots + 2001$

Sum of odd integers $= n^2 = (1001)^2$

23. *(c)* $y = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$

$= \frac{\sin^3 x}{\sin x + \cos x} + \frac{\cos^3 x}{\sin x + \cos x}$

$= \frac{\sin^3 x + \cos^3 x}{(\sin x + \cos x)}$

$= \sin^2 x + \cos^2 x - \sin x \cos x$

$y(x) = 1 - \frac{\sin 2x}{2}$

$y'(x) = 0 - \frac{\cos 2x}{x} \cdot 2$

$y'(x) = -\cos 2x$

24. *(a)* $16x^2 - 9y^2 - 64x + 18y - 90 = 0$

$= 16(x^2 - 4x) - 9(y^2 - 2y) = 90$

$= 16(x-2)^2 - 9(y-1)^2 = 90 + 16 \times 4 - 9 \times 1$

$= 16(x-2)^2 - 9(y-1)^2 = 145$

$= \frac{(x-2)^2}{\frac{145}{16}} - \frac{(y-1)^2}{\frac{145}{9}} = 1 \quad \dots\text{(i)}$

We know that, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots\text{(ii)}$

Foci $\Rightarrow (ae, 0)$

On comparing Eqs. (i) and (ii), we get

$\because \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$X = ae \Rightarrow x - 2$

$= + \sqrt{\frac{145}{16}} \times \frac{5}{3}$

$= \pm \frac{5\sqrt{145}}{12}$

$x = 2 \pm \frac{5\sqrt{145}}{12}$

$\Rightarrow \quad x = \frac{24 \pm 5\sqrt{145}}{12}$

$Y = 0 \Rightarrow y - 1 = 0 \Rightarrow y = 1$

Hence, $\left(\frac{24 \pm 5\sqrt{145}}{12}, 1\right)$

25. *(b)* $(a^2x^2 - 2ax + 1)^{51}$

For sum of coefficients put $x = 1$

$(a^2 - 2a + 1)^{51} = 0$

$\Rightarrow \quad \{(a-1)^2\}^{51} = 0$

$\Rightarrow \quad a = 1$

26. *(c)* Mean of the given data is

$\bar{x} = \frac{2 + 9 + 9 + 3 + 6 + 9 + 4}{7} = \frac{42}{7} = 6$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$ are

$2 - 6, 9 - 6, 9 - 6, 3 - 6, 6 - 6, 9 - 6, 4 - 6$

$\Rightarrow -4, 3, 3, -3, 0, 3, -2$

The absolute values of the deviations, i.e. $|x_i - \bar{x}|$ are 4, 3, 3, 3, 0, 3, 2.

The required mean deviation about the mean is

$\text{MD}(\bar{x}) = \frac{\sum_{i=1}^{7} |x_i - \bar{x}|}{7}$

$= \frac{4 + 3 + 3 + 3 + 0 + 3 + 2}{7}$

$= \frac{18}{7} = 2.57$

27. *(b)* Let n and p be the parameters of the binomial distribution.

Mean = 8 and variance = 4

$\Rightarrow \quad np = 8 \text{ and } npq = 4$

$\Rightarrow \quad q = \frac{1}{2} = p \text{ and } n = 16$

$\therefore$ Required probability $= P(X = 1)$

$= {}^{16}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{15} = 16 \times \left(\frac{1}{2}\right)^{16}$

$= \frac{2^4}{2^{16}} = \frac{1}{2^{12}}$

28. *(a)* The number of diagonals of a polygon with 15 sides is

$$= {}^nC_2 - n = {}^{15}C_2 - 15$$
$$= \frac{15 \times 14}{2} - 15$$
$$= 105 - 15 = 90$$

29. *(c)* Probability of Maths and Science students

$$= \frac{40}{100} = \frac{2}{5}$$

Probability of maths students $= \frac{60}{100} = \frac{3}{5}$

$$P(\text{Science/Maths}) = \frac{P(S \cap M)}{P(M)} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

30. *(b)* $x^2 + 2y^2 - 2x + 3y + 2 = 0$

$$= (x-1)^2 - 1 + 2\left(y^2 + \frac{3}{2}y + \frac{9}{16} - \frac{9}{16}\right) + 2 = 0$$
$$= (x-1)^2 + 2\left(y + \frac{3}{4}\right)^2 - \frac{9}{8} + 1 = 0$$
$$= (x-1)^2 + 2\left(y + \frac{3}{4}\right)^2 = \frac{1}{8}$$
$$= \frac{(x-1)^2}{\frac{1}{8}} + \frac{\left(y + \frac{3}{4}\right)^2}{\frac{1}{16}} = 1$$

Ellipse : $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{\frac{1}{16}}{\frac{1}{8}}}$

$$e = \sqrt{1 - \frac{8}{16}} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

31. *(c)* Mean of a set of observations

$$x_1, x_2, \ldots, x_{10} = 20$$

Then, according to question,

$$\frac{x_1 + 4 + x_2 + 8 + x_3 + 12 + \ldots + x_{10} + 40}{10}$$
$$= \frac{x_1 + x_2 + \ldots + x_{10}}{10} + \frac{4(1 + 2 + \ldots + 10)}{10}$$
$$= 20 + \frac{4 \times 55}{10} = 20 + \frac{220}{10}$$
$$= 20 + 22 = 42$$

32. *(c)* Probability of take a random from the word STATISTICS

$$= {}^{10}C_1$$

Probability of take a random from the word ASSISTANT

$$= {}^9C_1$$

The probability is that they are same letters T, A, I, S

$$= \frac{{}^3C_1 \times {}^3C_1 + {}^1C_1 \times {}^2C_1 + {}^2C_1 \times {}^1C_1 + {}^2C_1 \times {}^3C_1}{{}^{10}C_1 \times {}^9C_1}$$
$$= \frac{9 + 2 + 2 + 6}{90} = \frac{19}{90}$$

33. *(a)* $ax^2 + bx + c = 0$

Roots are $\cos\alpha$ and $\sin\alpha$

$\therefore \quad \cos\alpha \cdot \sin\alpha = \frac{c}{a}$ …(i)

and $\quad \cos\alpha + \sin\alpha = -\frac{b}{a}$ …(ii)

$\Rightarrow \quad (\cos\alpha + \sin\alpha)^2 = \frac{b^2}{a^2}$

Using Eq. (i), we get

$$\left(1 + 2\frac{c}{a}\right) = \frac{b^2}{a^2}$$

$\Rightarrow \quad a^2 - b^2 + 2ac = 0$

34. *(e)* Given, triangle of sides $= a, b, c$

$$= 4, 5, 6 \text{ cm}$$

$\therefore \quad S = \frac{a + b + c}{2}$

$$= \frac{4 + 5 + 6}{2} = \frac{15}{2}$$

Then, area of triangle

$$= \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{\frac{15}{2}\left(\frac{15}{2} - 4\right)\left(\frac{15}{2} - 5\right)\left(\frac{15}{2} - 6\right)}$$
$$= \sqrt{\frac{15}{2} \cdot \left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)} = \frac{15}{4}\sqrt{7}$$

35. *(b)*

10 → 6 Boys, 4 Girls

The team has atleast one boy

= Total case – No anyone boy

$$= {}^{10}C_4 - {}^6C_0$$
$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} - 1 = 210 - 1 = 209$$

36. *(b)* LOGARITHMS letters are 10.

A password is set with 3 distinct letters ${}^{10}C_3 \times 3!$

$$= \frac{10 \times 9 \times 8}{3 \times 2} \times 3 \times 2 = 720$$

37. *(b)* We know that,

$$5^4 = 625 = 13 \times 48 + 1$$

$\Rightarrow 5^4 = 13\lambda + 1$, where λ is a positive integer.

$$\Rightarrow (5^4)^{24} = (13\lambda + 1)^{24}$$

$$= {}^{24}C_0\,(13\lambda)^{24} + {}^{24}C_1\,(13\lambda)^{23} + {}^{24}C_2\,(13\lambda)^{22} + \ldots + {}^{24}C_{23}(13\lambda) + {}^{24}C_{24}$$

(by binomial theorem)

$$\Rightarrow 5^{96} = 13[{}^{24}C_0 13^{23}\lambda^{24} + {}^{24}C_1 13^{23}\lambda^{22} + \ldots + {}^{24}C_{23}\lambda] + 1$$

= (*a* multiple of 13) + 1

On multiplying both sides by 5, we get

$5^{97} = 5^{96} \cdot 5 = 5$ (a multiple of 13) + 5

Hence, the required remainder is 5.

38. *(a)* Total number of ways of assigning values 2, 3, 5 to $a, b, c, = 3! = 6$

Now, for quadratic equation $ax^2 + bx + c = 0$ to have real roots $b^2 - 4ac \geq 0$. This is possible only when $a = 2, b = 5, c = 3$ or $a = 3, b = 5, c = 2$

$\Rightarrow$ Required probability $= \frac{2}{6} = \frac{1}{3}$

39. *(d)* $\lim_{x \to \infty} \frac{3x^3 + 2x^2 - 7x + 9}{4x^3 + 9x - 2}$

$$= \lim_{x \to \infty} \frac{x^3\left[3 + \frac{2}{x} - \frac{7}{x^2} + \frac{9}{x^3}\right]}{x^3\left[4 + \frac{9}{x^2} - \frac{2}{x^3}\right]}$$

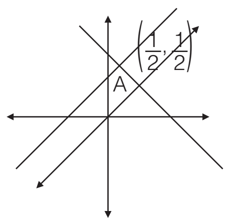
On putting $x \to \infty$, we get

$$= \frac{[3 + 0 - 0 + 0]}{[4 + 0 - 0]} = \frac{3}{4}$$

40. *(b)* we have,

$f(x) = \max\{x, 1 + x, 2 - x\}$

The graph of $f(x)$ is



Clearly from graph minimum value of $f(x)$ at point $A\left(\frac{1}{2}, \frac{3}{2}\right)$.

$\therefore$ Minimum value of $f(x)$ is $\frac{3}{2}$.

41. *(b)* We have equation of hyperbola is

$$xy + 3x - 2y - 10 = 0$$
$$xy + 3x - 2y - 6 = 4$$
$$(x - 2)(y + 3) = 4$$

We know that asymptote of hyperbola $xy = c$ is $x = 0$ and $y = 0$.

$\therefore$ Asymptote of hyperbola

$(x - 2)(y + 3) = 4$ is $x - 2 = 0,\ y + 3 = 0$

$\Rightarrow \quad x = 2,\ y = -3$

42. *(c)* $f(x) = x^6 + 6^x$

$f'(x) = 6x^5 + 6^x \log(6)$ $\quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

$\left[\because \frac{d}{dx}(a^x) = a^x \log(a)\right]$

43. *(a)* Given data 6, 5, 9, 13, 12, 8, 10

Mean of the given data $(\bar{x})$

$$= \frac{6 + 5 + 9 + 13 + 12 + 8 + 10}{7}$$

$$= \frac{63}{7} = 9$$

The deviation of the respective data from the mean i.e. $(x_i - \bar{x})$ are

$6 - 9, 5 - 9, 9 - 9, 13 - 9, 12 - 9, 8 - 9, 10 - 9$

$$(x_i - \bar{x}) = -3, -4, 0, 4, 3, -1, 1$$

$$(x_i - \bar{x})^2 = 9, 16, 0, 16, 9, 1, 1$$

$$\sum_{i=1}^{7}(x_i - \bar{x})^2 = 9 + 16 + 0 + 16 + 9 + 1 + 1$$

$$= 52$$

$\therefore$ Standard deviation (σ)

$$= \sqrt{\frac{1}{n}\sum_{i=1}^{7}(x_i - \bar{x})^2} = \sqrt{\frac{52}{7}}$$

44. *(a)* $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \to 0} \left\{\frac{2\sin^2 \frac{mx}{2}}{2\sin^2 \frac{nx}{2}}\right\}$

$$= \lim_{x \to 0} \left[\left\{\frac{\sin \frac{mx}{2}}{\frac{mx}{2}}\right\}^2 \cdot \frac{m^2x^2}{4} \cdot \frac{1}{\left\{\frac{\sin \frac{nx}{2}}{\frac{nx}{2}}\right\}^2} \cdot \frac{4}{n^2x^2}\right]$$

$$= \frac{m^2}{n^2} \times 1 = \frac{m^2}{n^2}$$

45. *(d)* $\lim_{x \to 0} \frac{\sqrt{1+2x}-1}{x}$

Using L' Hospital's Rule,

$$\lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+2x}} \cdot 2 - 0}{1}$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{1}{\sqrt{1+2x}}$$

Using limit, we get

$$= \frac{1}{\sqrt{1+2(0)}} = 1$$

46. *(b)* $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(3) = f'(g(3)) \cdot g'(3) \qquad \left[\because \begin{matrix} g(3) = 7 \\ g'(3) = 6 \end{matrix}\right]$$

$$= f'(7) \cdot 6$$

$$= 2 \times 6 = 12$$

47. *(d)* $\frac{\sqrt{3}}{\sin(20°)} - \frac{1}{\cos(20°)}$

$$= \frac{\sqrt{3}\cos(20°) - \sin(20°)}{\sin(20°)\cos(20°)}$$

$$= \frac{4\left[\frac{\sqrt{3}}{2}\cos(20°) - \frac{\sin(20°)}{2}\right]}{2\sin(20°)\cos(20°)}$$

$$[\because 2\sin A \cos A = \sin 2A]$$

$$= \frac{4(\sin 60° \cos 20° - \cos 60° \sin 20°)}{\sin 40°}$$

$$= \frac{4\sin(60° - 20°)}{\sin 40°}$$

$$[\because \sin(A-B) = \sin A \cos B - \cos A \sin B]$$

$$= \frac{4\sin 40°}{\sin 40°} = 4$$

48. *(a)* Given that,

$$P(X=1) = P(X=2)$$

$$= \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow \quad \lambda = 2$$

$$\therefore \quad P(X=6) = \frac{e^{-2}(2)^6}{6!}$$

$$= \frac{e^{-2} \times 4 \times 2^4}{6 \times 5 \times 4 \times 3 \times 2}$$

$$= \frac{4 \times e^{-2} \times 2^4}{45 \times 2^4} = \frac{4e^{-2}}{45}$$

49. *(d)* a and b are two consecutive number.

Let $a = n$, $b = n+1$

Now, $\sqrt{a^2 + b^2 + a^2b^2}$

$$= \sqrt{n^2 + (n+1)^2 + n^2(n+1)^2}$$

$$= \sqrt{n^2 + n^2 + 2n + 1 + n^2(n^2 + 2n + 1)}$$

$$= \sqrt{n^2 + n^2 + 2n + 1 + n^4 + 2n^3 + n^2}$$

$$= \sqrt{n^4 + n^2 + 1 + 2n^3 + 2n^2 + 2n + 1}$$

$$= \sqrt{(n^2 + n + 1)^2}$$

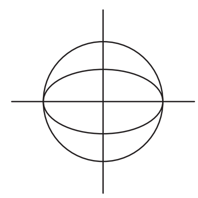
$$= n^2 + n + 1 = n(n+1) + 1$$

It is always odd.

$\because$ Probability of $\sqrt{a^2 + b^2 + a^2b^2}$ is an odd integer is 1.

50. *(b)* Given, $e = \frac{2\sqrt{2}}{3}$

$$e^2 = 1 - \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{b^2}{a^2} = 1 - \frac{8}{9} \qquad \ldots \text{(i)}$$



$$P(x) = \frac{\pi a^2 - \pi ab}{\pi a^2}$$

$$= 1 - \frac{b}{a}$$

$$= 1 - \frac{1}{3} \qquad \text{[using Eq. (i)]}$$

$$P(x) = \frac{2}{3}$$

51. *(c)* Given vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar.

Then, $\begin{vmatrix} 1 & 5 & 4 \\ 4 & 11 & m \\ 7 & 2 & 6 \end{vmatrix}$

$$\Rightarrow 1(66 - 2m) - 5(24 - 7m) + 4(8 - 77)$$

$$= 66 - 2m - 120 + 35m + 32 - 308$$

$$= 33m - 330 = 0$$

$$\Rightarrow \quad m = 10$$

52. *(a)* Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$$

Diagonals : $D_1 = \vec{a} + \vec{b}$ and $D_2 = \vec{b} + \vec{c}$

Area of parallelogram $= \frac{1}{2}|D_1 \times D_2|$ ...(i)

$$D_1 = \vec{a} + \vec{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$D_2 = \vec{b} + \vec{c} = 8\hat{i} + 12\hat{j} + 16\hat{k}$$

From Eq. (i),

$$|\text{Area}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}$$

$$= \frac{1}{2}|\hat{i}(64 - 72) + \hat{j}(48 - 32) + \hat{k}(24 - 32)|$$

$$= \frac{1}{2}|-8\hat{i} + 16\hat{j} - 8\hat{k}|$$

$$= \frac{1}{2}\sqrt{64 + 256 + 64} = \frac{1}{2}\cdot 8\sqrt{6} = 4\sqrt{6}$$

53. *(d)* $|\vec{a}| = 3, |\vec{b}| = 1, |\vec{c}| = 4,$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$(\vec{a} + \vec{b} + \vec{c})^2 = |a|^2 + |b|^2 + |c|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$\Rightarrow \quad 0 = (3)^2 + (1)^2 + (4)^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{26}{2} = -13$$

54. *(a)* $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$

$$|\vec{a} - \vec{b}|^2 = a^2 + b^2 - 2\vec{a}\cdot\vec{b}$$

$$1 = 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{1}{2} \Rightarrow \cos\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

55. *(b)* Given,

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$$

$$\vec{c} = \lambda\hat{i} + 9\hat{j} + \mu\hat{k}$$

Vectors are mutually orthogonal, so

$$\vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$$

$$\vec{a}\cdot\vec{b} = 2 - 4 + 2$$

$$= 2\lambda + 36 + \mu$$

$\Rightarrow \quad 2\lambda + 36 + \mu = 0$ ...(i)

$\lambda - 9 + 2\mu = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$\mu = 18, \lambda = -27$$

$$\therefore \quad \lambda + \mu = 18 - 27 = -9$$

56. *(d)* We have,

$$x^{\frac{2}{5}} + 3x^{\frac{1}{5}} - 4 = 0$$

Let $x^{\frac{1}{5}} = y$

$$y^2 + 3y - 4 = 0$$

$$\Rightarrow \quad y^2 + 4y - y - 4 = 0$$

$$\Rightarrow \quad y(y + 4) - 1(y + 4) = 0$$

$$\Rightarrow \quad (y + 4)(y - 1) = 0$$

$$\Rightarrow \quad y = -4, 1$$

$$\therefore \quad x^{\frac{1}{5}} = -4 \text{ or } x^{\frac{1}{5}} = 1$$

$$\Rightarrow \quad x = (-4)^5 \text{ or } x = 1$$

$$\Rightarrow \quad x = -1024 \text{ or } x = 1$$

57. *(c)* Given equations,

$$x^2 + ax + 1 = 0$$

$$x^2 - x - a = 0$$

$\because$ b is common root, so b satisfied both equations.

$$b^2 + ab + 1 = b^2 - b - a$$

$$= ab + b = -a - 1$$

$$\Rightarrow \quad b(a + 1) = -(a + 1)$$

$$\Rightarrow \quad b = -1$$

58. *(a)* Given, $\sin\theta - \cos\theta = 1$

$$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$$

$$= 1(1 + \sin\theta\cos\theta)$$

$= 1 + \sin\theta\cos\theta$ ...(i)

$[\because \sin\theta - \cos\theta = 1]$

On squaring both sides,

$$(\sin\theta - \cos\theta)^2 = (1)^2$$

$$\Rightarrow (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) = 1$$

$$\Rightarrow \quad 1 - 2\sin\theta\cos\theta = 1$$

$\Rightarrow \quad \sin\theta\cos\theta = 0$ ...(ii)

From Eqs. (ii) and (i), we get

$$\sin^3\theta - \cos^3\theta = 1$$

59. *(b)* Probability of sum of 7

= (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6)

and probability of sum of 11

= (6, 5), (5, 6)

$$P(x) = \frac{6}{36} + \frac{2}{36}$$

$$= \frac{8}{36} = \frac{2}{9}$$

60. *(a)* $\frac{1}{9!}+\frac{1}{3!\,7!}+\frac{1}{5!\,7!}+\frac{1}{7!\,3!}+\frac{1}{9!}$

$$=\frac{1}{10!}\left[\frac{10!}{9!1!}+\frac{10!}{3!\,7!}+\frac{10!}{5!\,7!}+\frac{10!}{7!\,3!}+\frac{10!}{9!\,1!}\right]$$

$$=\frac{1}{10!}[{}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9]$$

$$=\frac{1}{10!}\cdot 2^9=\frac{2^9}{10!}$$

61. *(a)* The given differential equation is

$$(y''')^2+(y'')^3-(y')^4+y^5=0$$

Clearly, its order is 3 and degree is 2.
Hence, option (a) is correct.

62. *(d)* Given that,

$$I=\int_{-2}^{2}|x|\,dx$$

$$=-\int_{-2}^{0}x\,dx+\int_0^2 dx$$

$$=-\left[\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}\right]_0^2$$

$$=-(-2)+(2)=4$$

63. *(b)* Given that,

$$I=\int_{-1}^{0}\frac{dx}{x^2+2x+2}$$

$$=\int_{-1}^{0}\frac{dx}{(x+1)^2+1}=[\tan^{-1}(x+1)]_{-1}^{0}$$

$$=[\tan^{-1}(1)-\tan^{-1}(0)]=\frac{\pi}{4}$$

64. *(e)* We know, $\int_2^4[3-f(x)]dx=7$

$\Rightarrow \int_2^4 3dx-\int_2^4 f(x)\,dx=7$

$\Rightarrow (3x)_2^4-\int_2^4 f(x)\,dx=7$

$\Rightarrow (12-6)-\int_2^4 f(x)\,dx=7$

$\Rightarrow 6-\int_2^4 f(x)\,dx=7$

$\Rightarrow \int_2^4 f(x)\,dx=-1$ ...(i)

Now, $\int_{-1}^{4} f(x)\,dx=4$

$\Rightarrow \int_{-1}^{2} f(x)dx+\int_2^4 f(x)dx=4$

$\Rightarrow \int_{-1}^{2} f(x)\,dx-1=4$ [from Eq. (i)]

$\Rightarrow \int_{-1}^{2} f(x)\,dx=5$

65. *(a)* Given that,

$$I=\int\frac{xe^x}{(1+x)^2}dx=\int\frac{(x+1-1)}{(1+x)^2}e^x\,dx$$

$$=\int e^x\left(\frac{1}{1+x}-\frac{1}{(1+x)^2}\right)dx$$

$$=\frac{e^x}{1+x}+C$$

66. *(a)* $2^{2000}=(2^4)^{500}$

$$=(16)^{500}=(17-1)^{500}$$

When divided by 17, then remainder $=(-1)^{500}=1$

Hence, remainder $=1$

67. *(b)* $T_{r+1}={}^8C_r x^{8-r}3^r$

For the coefficient of x^5,

$$8-r=5\Rightarrow r=3$$

$\therefore$ Coefficient of $x^5={}^8C_3\cdot 3^3$

$$=\frac{8!}{3!5!}\times 3^3=\frac{8\times7\times6}{6}\times 3^3$$

$$=8\times7\times3^3=56\times27$$

$$=1512$$

68. *(c)* $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$

$$=5\cos\theta+3[\cos\theta\cos 60^\circ-\sin\theta\sin 60^\circ]+3$$

$$=5\cos\theta+3\left[\frac{\cos\theta}{2}-\frac{\sqrt3}{2}\sin\theta\right]+3$$

$$=\frac{13}{2}\cos\theta-\frac{3\sqrt3}{2}\sin\theta+3$$

Let $\frac{13}{2}=a$ and $\frac{3\sqrt3}{2}=b$

Then expression becomes,

$$a\cos\theta-b\sin\theta+3$$

Maximum value of this type of expression is equal to

$[a^2+b^2]^{\frac12}+3=$ Maximum value

After putting values of a and b, we get

$[49]^{\frac12}+3=$ Max value

$10=$ Max value

69. *(c)* Let $z=x+iy$; $z+iz=(x-y)+i(x+y)$ and $iz=-y+ix$. If A is the area of triangle formed by z, $z+iz$ and iz, then

$$A=\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1 - R_3$

$$A=\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 0 \end{vmatrix}$$

$$=\frac{1}{2}(x^2+y^2) = \frac{1}{2}|z|^2$$

70. *(c)* $\because f(-x) = f(x)$

$$-f'(-x) = f'(x) \Rightarrow -f'(0) = f'(0)$$

$$\Rightarrow 2f'(0) = 0 \Rightarrow f'(0) = 0$$

71. *(c)* Here, $x_1 = 4, y_1 = -2, x_2 = 8, y_2 = 6$ and $m:n = 7:5$

$$\therefore x = \frac{mx_2 + nx_1}{m+n} = \frac{7\times 8 + 5\times 4}{12}$$

$$= \frac{56+20}{12} = \frac{76}{12} = \frac{19}{3}$$

and $$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{7\times 6 + 5\times(-2)}{7+5}$$

$$= \frac{42-10}{12} = \frac{32}{12} = \frac{8}{3}$$

$$\therefore (x, y) = \left(\frac{19}{3}, \frac{8}{3}\right)$$

72. *(d)* Area of triangle $= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix}$$

[Applying $c_2 \to c_2 + c_1$]

$$= \frac{a+b+c}{2}\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

73. *(d)* According to question,

$$\{x-(a+b)\}^2 + \{y-(b-a)\}^2$$

$$= \{x-(a-b)\}^2 + \{y-(a+b)\}^2$$

$$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$$

$$= x^2 + (a+b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$$

By solving, we get

$$\Rightarrow bx - ay = 0$$

74. *(b)* Given equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{...(i)}$$

$$\Rightarrow bx + ay = ab$$

$$\Rightarrow bx + ay - ab = 0$$

$$\therefore m = -\frac{b}{a}$$

So, equation of line passing through (a, b) and parallel to Eq. (i) is

$$y - b = -\frac{b}{a}(x-a)$$

$$ay - ab = -bx + ab$$

$$ay + bx = 2ab$$

$$\frac{y}{b} + \frac{x}{a} = 2$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

75. *(b)* Given, that

Area of triangle $= 18$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2a & a & 1 \\ a & 2a & 1 \\ a & a & 1 \end{vmatrix} = \pm 18$$

$$\Rightarrow \begin{vmatrix} 2a & a & 1 \\ a & 2a & 1 \\ a & a & 1 \end{vmatrix} = \pm 36$$

$$\Rightarrow 2a(2a-a) - a(a-a) + 1(a^2 - 2a^2) = \pm 36$$

$$\Rightarrow 2a^2 - a^2 = \pm 36$$

$$\Rightarrow a^2 = \pm 36$$

$$\Rightarrow a^2 = 36$$

$$\Rightarrow a = \pm 6$$

Now, centroid of the given triangle will be

$$= \left(\frac{2a+a+a}{3}, \frac{a+2a+a}{3}\right) = \left(\frac{4a}{3}, \frac{4a}{3}\right)$$

When $a = 6$, centroid $= \left(\frac{4\times 6}{3}, \frac{4\times 6}{3}\right) = (8, 8)$

76. *(c)* Let the coordinates of third vertex be (x, y).

Given that, area of a triangle = 5

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ x & y & 1 \end{vmatrix} = 5$$

$$\Rightarrow 2(-2-y) - 1(3-x) + 1(3y+2x) = 10$$

$$\Rightarrow -4 - 2y - 3 + x + 3y + 2x = 10$$

$$\Rightarrow \quad 3x + y = 17 \quad ...(i)$$

Since, third vertex lies as

$$y = x + 3 \quad ...(ii)$$

By solving Eqs. (i) and (ii), we get

$$x = \frac{7}{2}, \; y = \frac{13}{2}$$

77. *(b)* Given equation of pair of straight lines is

$x^2 + y^2 + 2gx + 2fy + 1 = 0$

Since, the necessary and sufficient condition for pair of straight lines is

$$\begin{vmatrix} a & h & g \\ h & b & f \\ h & f & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(1 - f^2) + g(0 - g) = 0$$

$$\Rightarrow \quad 1 - f^2 - g^2 = 0$$

$$\Rightarrow \quad f^2 + g^2 = 1$$

78. *(c)* Given equation of straight line is

$x^2 - 5xy + 4y^2 + 3x - 4 = 0$

$$\therefore \quad \tan\theta = \left| \frac{2\sqrt{\left(-\frac{5}{2}\right)^2 - 4}}{5} \right|$$

$$= \left| \frac{2\sqrt{\frac{25}{4} - 4}}{5} \right| = \frac{2}{5} \times \sqrt{\frac{9}{4}} = \frac{2}{5} \times \frac{3}{2} = \frac{3}{5}$$

$$\Rightarrow \quad \tan^2\theta = \frac{9}{25}$$

79. *(c)* Given that,

$$3\hat{i} + 2\hat{j} - 5\hat{k} = x(2\hat{i} - \hat{j} + \hat{k}) + y(\hat{i} + 3\hat{j} - 2\hat{k}) + 2(-2\hat{i} + \hat{j} - 3\hat{k})$$

$$\Rightarrow 3\hat{i} + 2\hat{j} - 5\hat{k} = i(2x + y - 2z) + \hat{j}(-x + 3y + z) + \hat{k}(x - 2y - 3z)$$

By equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$$\Rightarrow \quad 2x + y - 2z = 3 \quad ...(i)$$

$$-x + 3y + z = 2 \quad ...(ii)$$

$$x - 2y - 3z = -5 \quad ...(iii)$$

By solving Eqs. (i), (ii) and (iii), we get

$$x = 3, \; y = 1, \; z = 2$$

80. *(a)* $\because \sin 15° = \sin(45° - 30°)$

$$= \sin 45° \cos 30° - \sin 30° \cos 45°$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

81. *(b)* Since, $\vec{a}$ and $\vec{b}$ are collinear vector. Therefore,

$$\vec{a} = \lambda \vec{b} \quad ...(i)$$

$$\because \quad \vec{a} \cdot \vec{b} = 27$$

$$\Rightarrow \quad |\vec{a}||\vec{b}| \cos 0° = 27$$

$$\Rightarrow \quad |\vec{b}| \cdot \sqrt{9 + 36 + 36} = 27$$

$$\Rightarrow \quad |\vec{a}| = \frac{27}{9} = 3$$

By Eq. (i),

$$\vec{a} = \lambda \vec{b}$$

$$\Rightarrow \quad |\vec{a}| = |\lambda| |\vec{b}|$$

$$3 = |\lambda| \cdot 9$$

$$\Rightarrow \quad |\lambda| = \pm \frac{1}{3}$$

$$\therefore \quad \vec{a} = \pm \frac{1}{3}(3\hat{i} + 6\hat{j} + 6\hat{k})$$

$$\vec{a} = \pm(\hat{i} + 2\hat{j} + 2\hat{k})$$

82. *(e)* Given that,

$|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 30$

$$\because \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\Rightarrow \quad 30 = 13.5 \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{30}{13.5} = \frac{6}{13}$$

$$\Rightarrow \quad \sin^2\theta = 1 - \frac{36}{169}$$

$$\Rightarrow \quad \sin^2\theta = \frac{133}{169}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{133}}{13}$$

$$\therefore \quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$= 13.5 \cdot \frac{\sqrt{133}}{13}$$

$$= \frac{65}{13}\sqrt{133}$$

83. *(b)* Given that, ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800:1$

$$\frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$\Rightarrow \quad 56 \times 55 \times (51-r) = 30800$

$\Rightarrow \quad r = 41$

84. *(c)* Distance between parallel lines $y = 2x + 4$ or $2x - y + 4 = 0$ and $y = 2x - 1$ or $2x - y - 1 = 0$ is

$$= \frac{4+1}{\sqrt{(2)^2 + (1)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

85. *(c)* $({}^7C_0 + {}^7C_1) + ({}^7C_2 + {}^7C_3) + \ldots + ({}^7C_6 + {}^7C_7)$

$= {}^7C_0 + {}^7C_1 + \ldots + {}^7C_7$

$= 2^7 \; [\because C_0 + C_1 + C_2 + \ldots + C_n = 2^n]$

86. *(d)* $(1 - 3x + 7x^2)(1-x)^{16}$

$= (1 - 3x + 7x^2)$

$({}^{16}C_0 - {}^{16}C_1 x^1 + {}^{16}C_2 x^2 + \ldots + {}^{16}C_{16} x^{16})$

$= (1 - 3x + 7x^2)(1 - 16x + 120x^2 + \ldots)$

$\therefore$ Coefficient of $x = -16 - 3 = -19$

87. *(b)* Radius of circle is $\sqrt{(4-2)^2 + (5-2)^2} = \sqrt{13}$

So, equation of circle is

$$(x-2)^2 + (y-2)^2 = 13$$

$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 4 - 4y = 13$

$\Rightarrow \quad x^2 + y^2 - 4x - 4y - 5 = 0$

88. *(d)* Let the points are $A(2, 0, 3)$, $B(0, 3, 2)$ and $D(0, 0, 1)$.

We know that Z-coordinate of every point an xy-plane is zero so let $p(x, y, 0)$ be a point on xy-plane such that $PA = PB = PC$.

Now, $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-3)^2 + (0-2)$

$\Rightarrow \quad 4x - 6y = 0 \Rightarrow 2x - 3y = 0 \quad \ldots(\text{i})$

and, $\quad PB = PC$

$\Rightarrow \quad PB^2 = PC^2$

$\Rightarrow (x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$

$\Rightarrow \quad -6y + 12 = 0$

$\Rightarrow \quad y = 2 \quad \ldots(\text{ii})$

Putting $y = 2$ in Eq. (i), we get $x = 3$

Hence, the required point is (3, 2, 0).

89. *(a)* We have,

$f(1) = 2$ and $f(x+y) = f(x) \cdot f(y)$

Now, $f(2) = f(1+1) = f(1) \cdot f(1) = 2 \cdot 2 = 2^2$

$f(3) = f(2+1) + f(2) \cdot f(1) = 2^2 \cdot 2 = 2^3$

and so on

$\therefore \quad f(x) = 2^n \quad \ldots(\text{i})$

Now, we have

$$\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$$

$\Rightarrow f(a+1) + f(a+2) + \ldots + f(a+n) = 16(2^n - 1)$

$\Rightarrow f(a) \cdot f(1) + f(a) \cdot f(2) + \ldots + f(a) \cdot f(n) = 16(2^n - 1)$

$\Rightarrow f(a) = [f(1) + f(2) + \ldots + f(n)] = 16(2^n - 1)$

$\Rightarrow f(a)[2 + 2^2 + \ldots + 2^n] = 16(2^n - 1)]$

$\Rightarrow \quad f(a) \cdot \left[2\frac{(2^n - 1)}{2-1}\right] = 16(2^n - 1)$

$\Rightarrow \quad 2f(a) \cdot (2^n - 1) = 16 \cdot (2^n - 1) \Rightarrow f(a) = 8$

$\Rightarrow \quad 2^a = 8 \quad [\because f(x) = 2^n \Rightarrow f(a) = 2^a]$

$\Rightarrow \quad 2^a = 2^3 = a = 3$

90. *(d)* Given that,

${}^nC_{r-1} = 36, \; {}^nC_r = 84$

and $\quad {}^nC_{r+1} = 126$

Here, $\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{36}{84}$ and $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{84}{126}$

$\Rightarrow 3n - 10r = -3$ and $4n - 10r = 6$

By solving these equations, we get

$n = 9, \; r = 3$

91. *(b)* $\because f$ is continuous

$\therefore \quad f(0) = f(0+h) = f(0-h)$

$$f\left(\frac{1}{4}\right) = f\left(\frac{1}{4} + h\right)$$

$$f(0) = f\left(0 + \frac{1}{4}\right)$$

Given, $\quad f\left(\dfrac{1}{2}\right) = f\left(\dfrac{1}{2^2}\right)$

$\therefore \quad f(0) = f\left(0 + \dfrac{1}{2^2}\right) = \dfrac{1}{2} \quad \ldots(\text{i})$

Therefore, $4f\left(\dfrac{1}{4}\right)$

$= 4 \cdot \dfrac{1}{2} \quad$ [using Eq. (i)]

$= 2$

92. *(c)* $\lim_{x \to \infty} (\sqrt{x^2+1} - \sqrt{x^2-1})$

$$= \lim_{x \to \infty} (\sqrt{x^2+1} - \sqrt{x^2-1}) \frac{(\sqrt{x^2+1} + \sqrt{x^2-1})}{(\sqrt{x^2+1} + \sqrt{x^2-1})}$$

(by rationalization)

$$= \lim_{x \to \infty} \frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x \to \infty} \frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$= \lim_{x \to \infty} \frac{2}{x\left(\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x^2}}\right)} = 0$$

93. *(e)* $f'(1) = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$; function is differentiable.

$f(1) = 0$ and $\lim_{h \to 0} \frac{f(1+h)}{h} = 5$; Given function is continuous.

Hence, $f'(1) = \lim_{h \to 0} \frac{f(1+h)}{h} = 5$

94. *(d)* We have,

$$f(x) = 2x^3 - 15x^2 + 36x + 4$$

$$\Rightarrow \quad f'(x) = 6x^2 - 30x + 36$$

and $\quad f''(x) = 12x - 30$

At point of local maximum as minimum, we must have

$$f'(x) = 0 \Rightarrow 6(x^2 - 5x + 6) = 0 \Rightarrow x = 2, 3$$

Clearly, $f''(2) = 24 - 30 = -6 < 0$

and $\quad f''(3) = 36 - 30 = 6 > 0$

So, $f(x)$ has local maximum at $x = 2$.

95. *(c)* We have,

$$\int f(x)\cos x\,dx = \frac{1}{2}\{f(x)\}^2 + C$$

On differentiating both sides, we get

$$f(x)\cos x = f(x) f'(x)$$

$$\because \quad f'(x) = \cos x$$

$$\Rightarrow \quad f(x) = \sin x + C$$

$$\because \quad f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + C = 1 + C$$

96. *(a, e)* Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}{1+\sin\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(\pi - x)dx}{1+\sin x} \quad \text{...(ii)}$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

By adding Eqs. (i) and (ii), we get

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi\,dx}{1+\sin x}$$

$$\Rightarrow \quad 2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx$$

$$\Rightarrow \quad 2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} [\sec^2 x - \sec x \tan x]\,dx$$

$$\Rightarrow \quad 2I = \pi[\tan x - \sec x]_{\pi/4}^{3\pi/4}$$

$$\Rightarrow \quad 2I = \pi[-1-(-\sqrt{2}) - (1-\sqrt{2})]$$

$$\Rightarrow \quad 2I = \pi[-1+\sqrt{2}-1+\sqrt{2}]$$

$$\Rightarrow \quad 2I = \pi[-2+2\sqrt{2}]$$

$$\therefore \quad I = \pi[\sqrt{2}-1]$$

$$= \frac{\pi(\sqrt{2}-1)}{(\sqrt{2}+1)}(\sqrt{2}+1)$$

$$= \frac{\pi}{\sqrt{2}+1}$$

97. *(c)* Let $I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\,dx \quad \text{...(i)}$

$$\Rightarrow \quad I = \int_2^{\frac{\pi}{2}} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)} + 2^{\cos\left(\frac{\pi}{2}-x\right)}}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} \quad \text{...(ii)}$$

By adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x} + 2^{\cos x}}{2^{\sin x} + 2^{\cos x}}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx = [x]_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \frac{\pi}{2}$$

$$\Rightarrow \quad I = \frac{\pi}{4}$$

98. *(d)* $\lim_{x \to 0} \dfrac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^2}$

where, $f(0) = 0$, $g(0) = 0$

$\therefore \quad I = \lim_{x \to 0} \dfrac{f'(x)}{g'(x)}$

where, $f'(x) = \sin\sqrt{x^2}\, \dfrac{d}{dx}(x^2) - 0$

$= 2x \sin x$

$\therefore \quad I = \lim_{x \to 0} \dfrac{2x \sin x}{2x}$

$= \lim_{x \to 0} \sin x = 0$

99. *(b)* Required area $= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x\, dx$

$= \int_{\frac{\pi}{2}}^{\pi} \left[\dfrac{1 - \cos 2x}{2}\right] dx$

$= \dfrac{1}{2} \int_{\frac{\pi}{2}}^{\pi} (1 - \cos 2x)\, dx$

$= \dfrac{1}{2}\left[x - \dfrac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= \dfrac{1}{2}\left[(\pi - 0) - \left(\dfrac{\pi}{2} - 0\right)\right]$

$= \dfrac{1}{2}\left[\dfrac{\pi}{2}\right] = \dfrac{\pi}{4}$

100. *(a)* Given, system of equation is

$y = e^x (A \cos x + B \sin x)$.

$\Rightarrow \quad \dfrac{dy}{dx} = e^x(-A \cos x + B \cos x) + y \qquad \text{...(i)}$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = e^x(-A \sin x + B \cos x) + e^x$

$[-A \cos x - B \sin x] + \dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx} - y\right) y + \dfrac{dy}{dx} \qquad$ [by Eq. (i)]

$\Rightarrow \quad \dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

$\Rightarrow \quad y'' - 2y' + 2y = 0$

This is required differential equation.

101. *(*)* $(i - \sqrt{3})^{13}$

$= 2^{13} \times i^{13} \left[\dfrac{1 + \sqrt{3}i}{2}\right]^{13}$

$= 2^{13} i^{13} (-1)^{13} \left[\dfrac{-1 - \sqrt{3}i}{2}\right]^{13}$

$= -2^{13} \cdot 1^{13} w^{13} = -2^{13} \cdot i \cdot \left[\dfrac{-1 + \sqrt{3}i}{2}\right]$

$= -2^{13}[-i - \sqrt{3}] = -i\,2^{13} + 2^{13}\sqrt{3}$

Hence, real part is $2^{13}\sqrt{3}$.

102. *(b)* $\lim_{x \to 0} \dfrac{1 + x - e^x}{x^2}$

$= \lim_{x \to 0} \dfrac{1 - e^x}{2x}$ [by L' Hospital's rule]

$= \lim_{x \to 0} \dfrac{-e^x}{2} = -\dfrac{e^0}{2} = -\dfrac{1}{2}$

103. *(b)* We have,

$I = \int \dfrac{(\sin x + \cos x)(2 - \sin 2x)}{\sin^2 2x} dx$

Put $\sin x - \cos x = t \Rightarrow (\sin x + \cos x)dx = dt$

and $(\sin x - \cos x)^2 = t^2 \Rightarrow 1 - \sin 2x = t^2$

$\Rightarrow \quad \sin 2x = 1 - t^2$

$\therefore \quad I = \int \dfrac{(2 - (1 - t^2))dt}{(1 - t^2)^2}$

$\Rightarrow \quad I = \int \dfrac{(1 + t^2)dt}{(1 - t^2)^2}$

$\Rightarrow \quad I = \int \dfrac{1 + t^2}{1 - 2t^2 + t^4} dt$

$\Rightarrow \quad I = \int \dfrac{1 + 1/t^2}{\frac{1}{t^2} + t^2 - 2} dt$

$\Rightarrow \quad I = \int \dfrac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2} dt$

Put $t - \dfrac{1}{t} = y \Rightarrow \left(1 + \dfrac{1}{t^2}\right)dt = dy$

$\therefore \quad I = \int \dfrac{dy}{y^2} = -\dfrac{1}{y} + C$

$\Rightarrow \quad I = \dfrac{-1}{t - \frac{1}{t}} + C$

$\Rightarrow \quad I = \dfrac{t}{1 - t^2} + C$

$\Rightarrow \quad I = \dfrac{\sin x - \cos x}{\sin 2x} + C$

104. *(c)* Equation of plane whose distance from origin is P and normal is $\hat{n}$ is

$P = \vec{r} \cdot \hat{n}$

Given that, $P = 5$

$$\therefore \quad \hat{n} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}}$$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$$

By formula,

$$5 = \vec{r} \cdot \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$$

$$\Rightarrow \vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 15$$

105. *(d)* Given that, $\dfrac{\sin A - \sin B}{\cos A + \cos B}$

$$= \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)} = \tan\left(\frac{A-B}{2}\right)$$

106. *(b)* Given that,

$$x = A\cos 4t + B\sin 4t \quad \text{...(i)}$$

Differentiating w.r.t to t,

$$\frac{dx}{dt} = 4 \cdot A(-\sin 4t) + 4 \cdot B\cos 4t$$

$$\Rightarrow \frac{dx}{dt} = 4[-A\sin 4t + B\cos 4t]$$

Again differentiating w.r.t to t

$$\Rightarrow \frac{d^2x}{dt^2} = 4[-4 \cdot A\cos 4t + (-4)B\sin 4t]$$

$$= -16[A\cos 4t + B\sin 4t]$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16x \quad \text{[by Eq. (i)]}$$

107. *(a)* $\because (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 \cdot x^2 + \ldots + {}^nC_n \cdot x^n$

Take $x = 1$

$$(1+1)^n = {}^nC_0 + {}^nC_1 \cdot (1) + {}^nC_2 \cdot (1)^2 + \ldots + {}^nC_n \cdot (1)^n$$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$$

Now, arithmetic mean

$$\overline{X} = \frac{\Sigma_{i=1}^n x_i}{N}, \text{ where } N = (n+1)$$

$$\Rightarrow \overline{X} = \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n}{n+1}$$

$$\Rightarrow \overline{X} = \frac{2^n}{n+1}$$

108. *(d)* Since, variance of first n natural number is

$$(\text{S.D.})^2 = \frac{n^2 - 1}{12}$$

$\therefore$ Variance of first 20 natural number is

$$(\text{S.D.})^2 = \frac{(20)^2 - 1}{12}$$

$$= \frac{400 - 1}{12}$$

$$= \frac{399}{12} = \frac{133}{4}$$

109. *(b)* Total numbers of elements in the set A = The selection of two distinct elements from given 10 elements.

$$\Rightarrow \quad n(A) = {}^{10}C_1 \times {}^9C_1 = 10 \times 9 = 90$$

110. *(b)* $P(E_1) = \dfrac{1}{2}$ and $P(E_2) = \dfrac{1}{6}$

So, required probability $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{6}\right) = \dfrac{1}{12}$

111. *(e)* $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$$\Rightarrow |A| = 0(2-3) - 1(1-9) + 2(1-6) = 8 - 10 = -2$$

$\therefore \quad C_{11} = (2-3) = -1, C_{12} = -(1-9) = 8$

$C_{13} = (1-6) = -5, C_{21} = -(1-2) = 1$

$C_{22} = (0-6) = -6, C_{23} = -(0-3) = -3$

$C_{31} = (3-4) = -1, C_{32} = -(0-2) = 2$

$C_{33} = (0-1) = -1$

$$\therefore \text{adj}|A| = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}[A]}{|A|} = \frac{\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}}{-2}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 4 & \frac{5}{2} \\ -\frac{1}{2} & 3 & -\frac{3}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix}$$

$\therefore$ Sum of all diagonal entries of A^{-1}

$$= \frac{1}{2} + 3 + \frac{1}{2} = 4$$

112. *(a)* Given, $f(x)=\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

$$=\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

[applying $R_1 \to R_1 + R_2 + R_3$]

$$=(x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

$$=(x+9)\begin{vmatrix} 0 & 0 & 1 \\ 2-x & x-2 & 2 \\ 1 & 6-x & x \end{vmatrix}$$

[applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$$=(x+9)[(2-x)(6-x)-(x-2)]$$
$$=(x+9)(x-2)[x-6-1]$$
$$f(x)=(x+9)(x-2)(x-7)$$

at $f(x)=0$

$(x+9)(x-2)(x-7)=0$

$\Rightarrow$ $x=-9, 2, 7$

Hence, other roots are 2 and 7.

113. *(a, d)* Given that, $[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}=0$

$$\Rightarrow \begin{bmatrix} 1+2x+15 \\ 3+5x+3 \\ 2+x+2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix} 2x+16 \\ 5x+6 \\ x+4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}=0$$

$\Rightarrow 2x+16+2(5x+6)+x(x+4)=0$

$\Rightarrow 2x+16+10x+12+x^2+4x=0$

$\Rightarrow x^2+16x+28=0$

$\Rightarrow x^2+2x+14x+28=0$

$\Rightarrow x(x+2)+14(x+2)=0$

$\Rightarrow (x+2)(x+14)=0$

$\Rightarrow x=-2, -14$

114. *(b)* We have,

$$A=\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$$

$$A^{-1}=\frac{1}{2x^2}\begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix}$$

$$\left\{\because A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right\}$$

$$\Rightarrow \quad A^{-1}=\begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix}$$

Now, it is given that

$$A^{-1}=\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\therefore \quad x=\frac{1}{2}$$

115. *(b)* Given that,

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}=ax^4+bx^3+cx^2+dx+e$$

$\Rightarrow x(6x-6x)-2(6x^2-6x)+x(x^3-x^2)$

$$=ax^4+bx^3+cx^2+dx+e$$

$\Rightarrow -12x^2+12x+x^4-x^3=ax^4+bx^3+cx^2+dx+e$

$\Rightarrow \quad x^4-x^3-12x^2+12x=ax^4+bx^3+cx^2+dx+e$

On equating the coefficient of both sides, we get

$a=1, b=-1, c=-12, d=12, e=0$

$\therefore 5a+4b+3c+2d+e=5\times1+4\times(-1)+3(-12)$
$+2(12)+0$
$=5-4-36+24$
$=29-40=-11$

116. *(b)* Given that,

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}=\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

[applying $C_3 \to C_3 + C_2$]

$$=(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$=(a+b+c)\times 0$ [$\because C_1$ and C_3 are equal]

$=0$

117. *(a)* Given,

$$f(x)=\begin{vmatrix}1 & 1 & 1\\ 2x & x-1 & x\\ 3x(x-1) & (x-1)(x-2) & x(x-1)\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 1\\ x+1 & -1 & x\\ 2(x+1)(x-1) & -2(x-1) & x(x-1)\end{vmatrix}$$

[applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$$=(x-1)\begin{vmatrix}0 & 0 & 1\\ (x+1) & -1 & x\\ 2(x+1) & -2 & x\end{vmatrix}$$

$$=(x-1)[-2(x+1)+2(x+1)]$$

$\Rightarrow\ f(x)=0$

$\therefore\ f(50)=0$

118. *(a)* Given,

$$\Delta(x)=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 1+\sin x & \cos x & 1+\sin x-\cos x\\ \sin x & \sin x & 1\end{vmatrix}$$

$$=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 0 & -\sin x & \sin x-1\\ \sin x & \sin x & 1\end{vmatrix}$$

[applying $R_1 \to R_2-(R_1+R_3)$]

$$=\begin{vmatrix}1 & \cos x & 1\\ 0 & -\sin x & -1\\ \sin x & \sin x & 1+\sin x\end{vmatrix}$$

[applying $C_3 \to C_3 + C_2$]

$$=1(0+\sin^2 x)+1(\sin x-\sin x\cos x)+(1+\sin x)(-\sin x-0)$$

$$=\sin^2 x+\sin x-\sin x\cos x-\sin x-\sin^2 x$$

$$=-\sin x\cos x$$

$$\Delta x=-\frac{\sin 2x}{2}$$

$$\therefore\ \int_0^{\frac{\pi}{2}}\Delta(x)dx=-\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 2x\,dx$$

$$=-\frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{4}[\cos\pi-\cos 0]$$

$$=\frac{1}{4}[-1-1]$$

$$=-\frac{2}{4}=-\frac{1}{2}$$

119. *(b)* Given that,

$x_1=1,\ y_1=2,\ z_1=3$

$x_2=-1,\ y_2=4,\ z_2=2$

and $x_3=3,\ y_3=1,\ z_3=1$

Equation of plane passing through these points is

$$\begin{vmatrix}x-1 & y-2 & z-3\\ -2 & 2 & -1\\ 2 & -1 & -2\end{vmatrix}=0$$

$$\Rightarrow (x-1)(-4-1)-(y-2)(y+2)+(z-3)(2-4)=0$$

$$\Rightarrow (x-1)(-5)-(y-2)(6)+(z-3)(-2)=0$$

$$\Rightarrow -5x+5-6y+12-2z+6=0$$

$$\Rightarrow -5x-6y-2z+23=0$$

$$\Rightarrow 5x+6y+2z-23=0$$

$$\Rightarrow 5x+6y+2z=23$$

120. *(a)* Let a be the first term of an AP and d is the common difference.

$\therefore\ a_k=a+(n-1)d$

Since, $a_k=5k+1$

$a+(k-1)d=5(k-1)+6$

$\Rightarrow\ a+(k-1)d=6+(k-1)6$

Equating both sides, we get

$a=6$ and $d=5$

$$\therefore\ S_{100}=\frac{n}{2}[2a+(n-1)d]$$

$$=\frac{100}{2}[2\times 6+99\times 5]$$

$$=50[12+495]=50(507)$$